

Persistent Visibility Reconstruction

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Talk Outline

1. Visibility Graphs of Staircase Polygons
2. Persistent Graphs. From Visibility Graphs to Persistent Graphs.
3. Our Research Problem
4. Results

Persistent Visibility
Reconstruction

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Outline

Staircase
Polygons' Visibility
Graphs

Persistent Graphs
Balanced Tableaux

Project Description

Summer Results
Definitions
Main Theorem
Proof Sketch

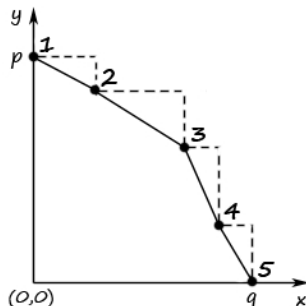
The End

Staircase Polygon

Definition

A **staircase polygon** is a staircase path together with the segments from the origin to p and the origin to q .

Example



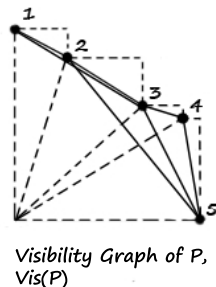
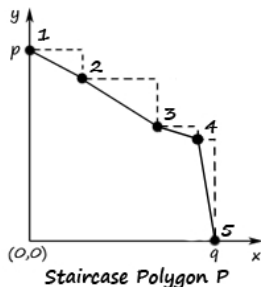
Staircase Polygon P

Visibility Graphs

Definition

The **visibility graph**, $Vis(P)$, of a simple polygon P is the graph with $V(Vis(P)) = Vertices(P)$ and whose edges correspond to internally visible pairs of vertices in P .

Example



Persistent Graphs

Example

1											
1	2										
0>	1	3									
0>	1*	1	4								
1	1*	0	1	5							
1	0>	0	0	1	6						
0>	0>	0	0	0	1	7					
1	1	0^	0^	1*	1*	1	8				
1	1	0^	1*	1*	0	0	1	9			
1	1	1	1*	0>	0	0	0	1	10		
1	1	1	1	1	0^	0^	1	1*	1	11	

Persistent Matrix M

Theorem (AEK, 1995)

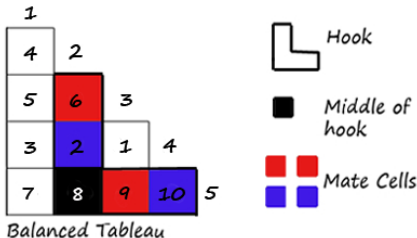
If P is a staircase polygon, then $\text{Vis}(P)$ is a persistent graph.

Balanced Tableaux

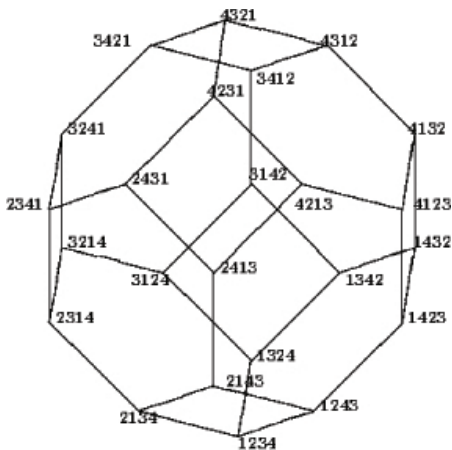
Definition

A **balanced tableau** is a Ferrer's diagram of staircase shape where each cell contains a distinct value between 1 and $\binom{n}{2}$ that satisfies a balanced property.

Example



Balanced tableaux $\leftrightarrow$ maximal chains in the Weak Bruhat Order of the Symmetric Group S_n .



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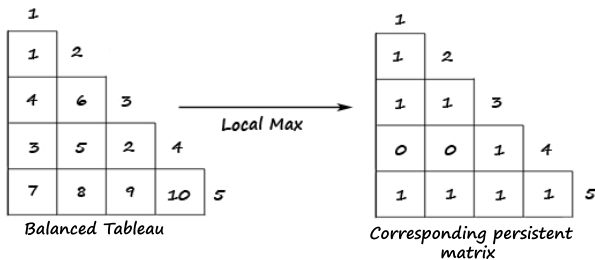
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Theorem (AEK,1995)

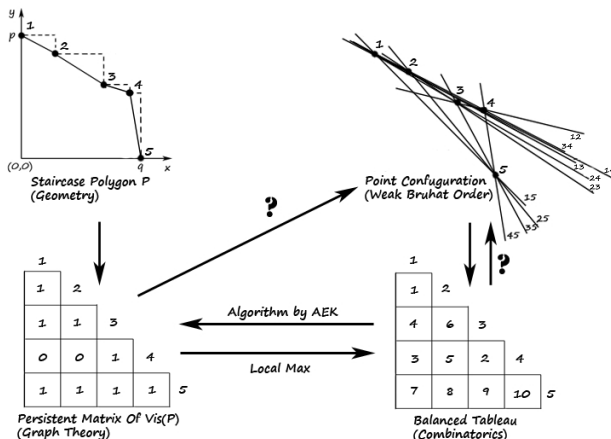
Persistent graphs are obtained precisely from balanced tableaux via local maximums (or dually by local minimums).



Our Project

The question: Is every persistent graph the visibility graph of a staircase polygon?

If true, then: Those graphs obtained from balanced tableaux via local max are precisely visibility graphs of staircase polygons.



Main Ideas

1. Persistent graphs can be decomposed into special subgraphs called factors.
2. Solve the reconstruction problem for factors.
3. Compose the factors solutions.

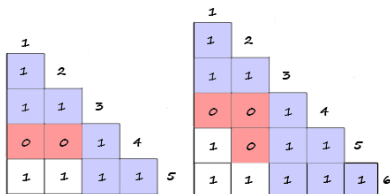
Factors

Definition

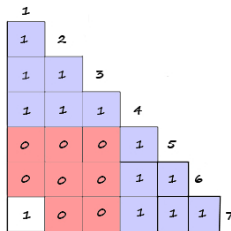
A vertex that blocks the visibility of two other vertices is called a **blocking vertex**.

Definition

A **factor** M_i is a persistent graph with only one blocking vertex, v_i .



Factors with blocking vertex 3



Factor with blocking vertex 4

Definition

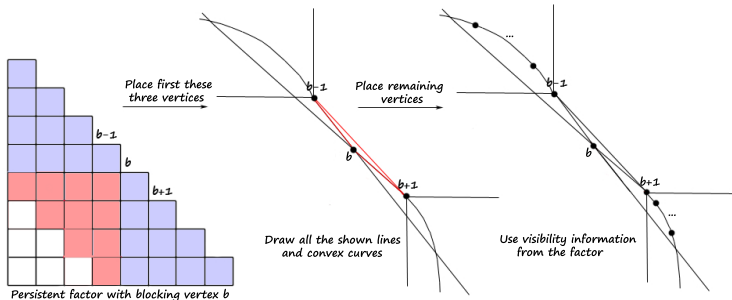
A **clique** is the persistent matrix in which every entry is 1.

Main Result

Theorem (Abello, Jimenez)

If M_i is a persistent factor, then M_i is the $\text{Vis}(P)$ of a staircase polygon P .

Reconstruction Algorithm (complexity $O(n^2)$)



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1. J. Abello, O. Egecioglu, K. Kumar, "Visibility Graphs of Staircase Polygons and the Weak Bruhat Order I: From Visibility Graphs to Maximal Chains" , Discrete and Computational Geometry, Vol. 14, No 3, 1995, pp 331-358.

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QUESTIONS
Thank you for listening!