Optimal Allocation And Scheduling Of Inspection Operations

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DIMACS

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Motivation

- Large volume of cargo shipped internationally
- Too costly to inspect all
- Need for increased security
  - Installation of wide range of sensor
- The US initiated the Container Security Initiative (CSI)

Background

- Various sensors to detect different restricted materials
- Restricted materials are classified into *j* risk categories
  - Nuclear material
  - Drugs
  - Hazardous materials
  - Other illegal substances

Background

x=0,1 - true status of container being acceptable, unacceptable

Multiple Risk Categories – $x = (x_1, x_2, \dots, x_k)$

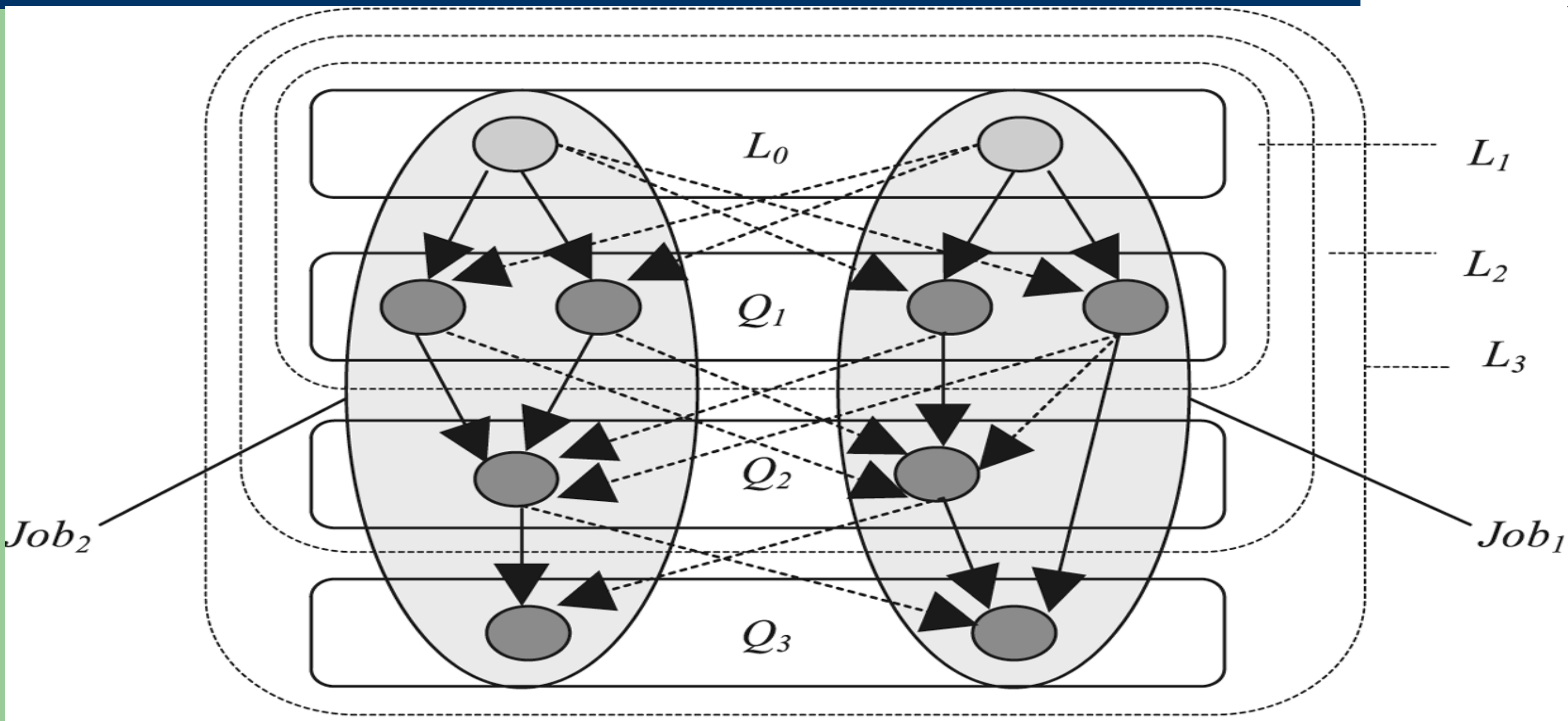
A vector of the true statuses w.r.t. *k* risk categories

Profile the containers based on

- Origin
- Destination
- Shipping fidelity
- Intelligence

Probability of risk category being present - $\overline{\pi} = (\pi_1, \pi_2, \dots, \pi_k)$

Open Shop Scheduling Problem



http://www.emeraldinsight.com/journals.htm?articleid=875958&show=html

Young's Work

- Find the optimal allocation strategy
  - Greatest effect on false acceptance probabilities
  - Weigh the effects on
    - optimal schedule
    - tardiness

Formulation

$$\min\ (1-w_T)\sum_{i=1}^{n}\sum_{j=1}^{k}\pi_{ij}[1-PTR_j y_{ij}]\prod_{s\neq j,s=1}^{k}[1-PFR_s y_{is}]+w_T\sum_{i=1}^{n}T_i$$

w_T weight of tardiness

i container i $(i=1,2,\ldots n)$

j inspection station j $(j=1,2,..k)$

π_{ij} prob. of risk category j present in cont. i

PTR_j prob. of true reject at station j

$y_{ij} =$ 1 if cont. i is inspected by station j

0 otherwise

PFR_j prob. of false reject at station j

T_i tardiness of cont. i

Constraints

s.t. $s_{ij} + t_{ij}y_{ij} - d_i - T_i \leq 0 \quad \forall i = 1,2,\ldots n; \forall j = 1,2,\ldots k$

$$s_{ij} + t_{ij}y_{ij} - M(1 - z_{igj}) - M(1 - y_{gj}) - s_{gj} \leq 0 \quad \forall i \neq g, \forall j = 1,2,\ldots k$$

$$s_{ij} + t_{ij}y_{ij} - M(1 - zz_{jhi}) - M(1 - y_{ih}) - s_{ih} \leq 0 \quad \forall i = 1,2,\ldots n, \forall j \neq h$$

s_{ij} **start time of inspection of cont.** i **at station** j

t_{ij} **processing time of cont.** i **at station** j

d_i **due time of cont.** i

w_T weight of tardiness

i container i $(i = 1,2,\ldots n)$

j inspection station j $(j = 1,2,..k)$

π_{ij} prob. of risk category j present in cont. i

M **a large positive number**

z_{igj} = **1 if cont.** i **precedes cont.** g **on station** j
0 otherwise

zz_{jhi} = **1 if station** j **precedes station** h **in inspection of cont.** i
0 otherwise

PTR_j prob. of true reject at station j

y_{ij} = 1 if cont. i is inspected by station j
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PFR_j prob. of false reject at station j

T_i tardiness of cont. i

Constraints

$$s_{ij} + t_{ij} y_{ij} - d_i - T_i \le 0 \;\; \forall i = 1,2,\ldots n; \forall j = 1,2,\ldots k$$

$$\Rightarrow s_{ij} + t_{ij} y_{ij} \le d_i + T_i$$

s_{ij}	**start time of inspection of cont.** i **at station** j	M	**a large positive number**
t_{ij}	**processing time of cont.** i **at station** j	z_{igj} =	**1 if cont.** i **precedes cont.** g **on station** j; **0 otherwise**
d_i	**due time of cont.** i	zz_{jhi} =	**1 if station** j **precedes station** h **in inspection of cont.** i; **0 otherwise**
w_T	weight of tardiness	PTR_j	prob. of true reject at station j
i	container i ($i = 1,2,\ldots n$)	y_{ij} =	1 if cont. i is inspected by station j; 0 otherwise
j	inspection station j ($j = 1,2,..k$)	PFR_j	prob. of false reject at station j
π_{ij}	prob. of risk category j present in cont. i	T_i	tardiness of cont. i

Constraints

$$s_{ij} + t_{ij} y_{ij} - M(1 - z_{igj}) - M(1 - y_{gj}) - s_{gj} \le 0 \ \forall i \ne g, \forall j = 1,2,\ldots k$$

$$\Rightarrow s_{ij} + t_{ij} y_{ij} \le s_{gj}$$

$$\Rightarrow s_{ij} + t_{ij} y_{ij} - \infty \le s_{gj}$$

s_{ij} **start time of inspection of cont.** i **at station** j

t_{ij} **processing time of cont.** i **at station** j

d_i **due time of cont.** i

w_T weight of tardiness

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$$\Rightarrow s_{ij} + t_{ij} y_{ij} \le s_{ih}$$

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Young's Conclusion

- Young's solution worked well for small n, k
  - Algorithm completely enumerated to find optimal schedule
  - Ex: n=10, k=2
    - 20 i,j's
    - y_{ij}={0,1}, so 2^{20} combinations
    - Other decision variables: z_{igj}, zz_{jhi}

Our Goals

- Find optimal allocation for large
  - Number of containers
  - Number of stations
- Possible Approaches
  - Revise Young's algorithm to allow for large numbers
  - Revise Young's formulation
  - Create new formulation

Thank you!

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