Introduction to local computation algorithms

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REU 2011

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Some motivation

The output can be too big to be displayed, so we want to ask for its parts only if we need them.

If we want to compute the solution localy, there must be some valid solution.

Some (de)motivation examples

Some examples for start:

- $a, b \in \{0,1\}^n$ find i-th digit of $a \cdot b$ (product done binary)
- $a, b \in \{0,1,2,3,4\}^n$ find i-th digit of $a + b$ (sum done decimal)

$$1232032123423242421023431114341003431242 31 + 24221243011212424124240323321231212 1212$$

query: 17 26
answer: 8 3

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1232032123423242**4**2102343**1**11434100343124231 +
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query: 17	26
answer: 8	3

Error correcting codes

Let m be a message coded by some ECC C (say repetition code), from $C(m)$ return m_i

message m:

1	0	1	0

1	1	1	0	0	0	1	1	1	0	0	0
1	0	1	0	0	1	1	1	1	0	1	0

Local computation algorithm

Let $F(x) = \{y | y \textit{ is valid solution for input } x\}$, $n = |x|$.
Local algorithm $\mathcal{A}$ is (randomized) algorithm which implements query access to an arbitrary $y \in F(x)$.
For each query i it produces y_i (the i-th bit of output).
Computation must be fast and consistent (i.e. there must exist a solution $y \in F(x)$ with all bits consistent with all previous answers).
BUT: we cannot remeber our previous answers, because of lack of space!

Our local problem

- Task: color a cubic graph G with “sufficiently” many colors
- do not construct a coloring of the whole graph, but in each query we ask for a color of one particular vertex

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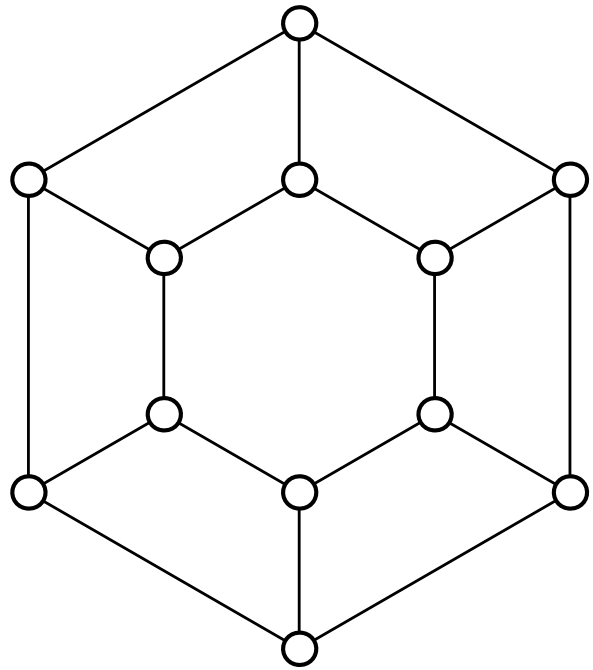

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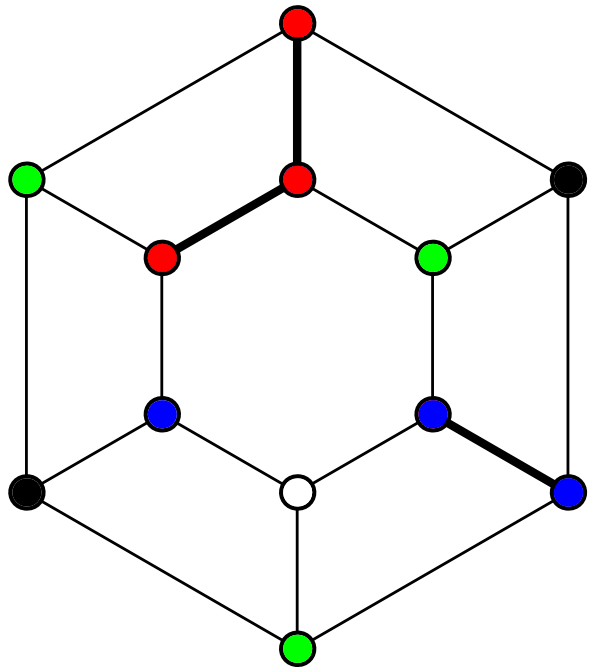

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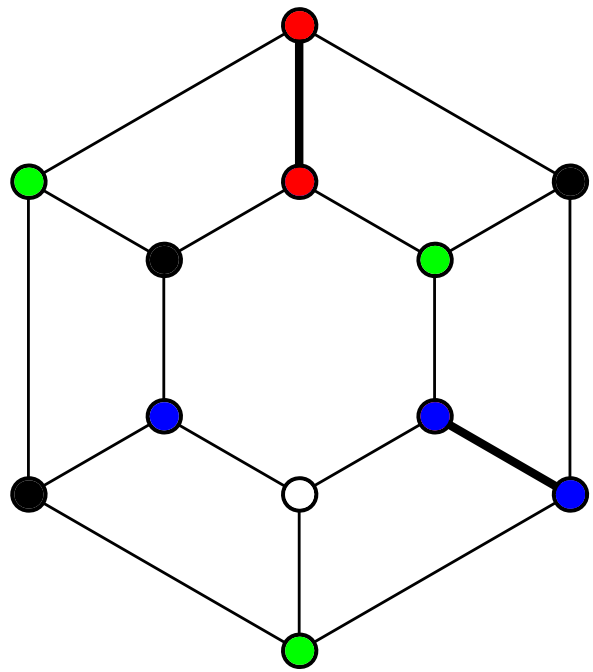

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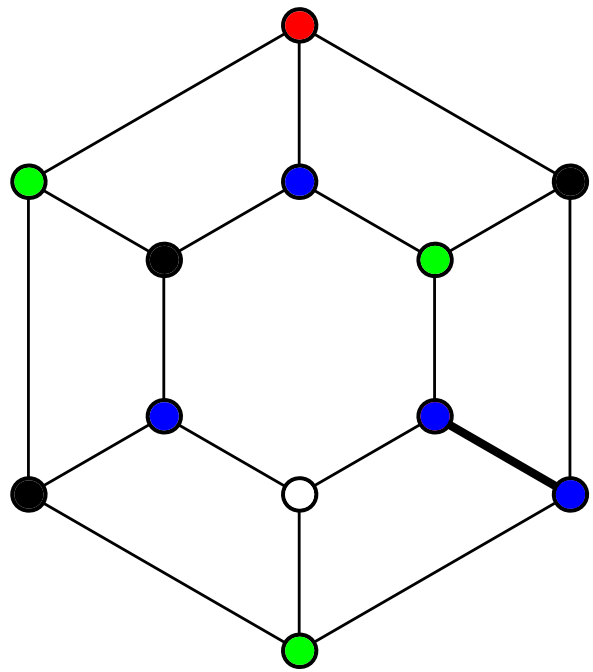

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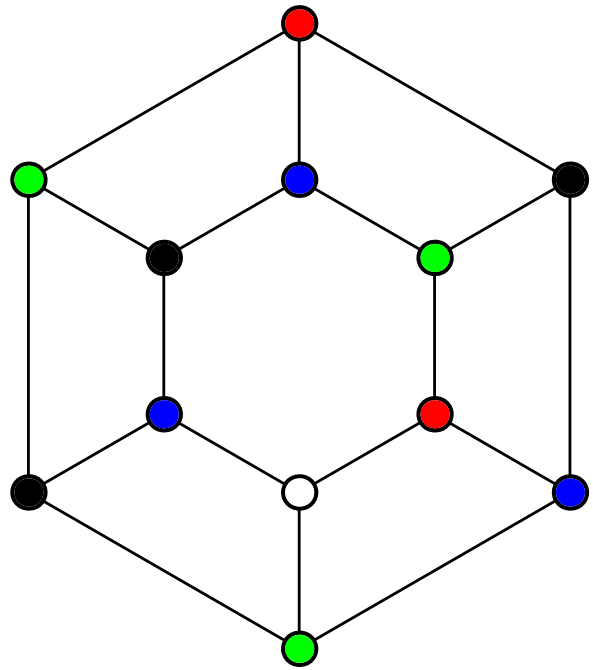

Last comments

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