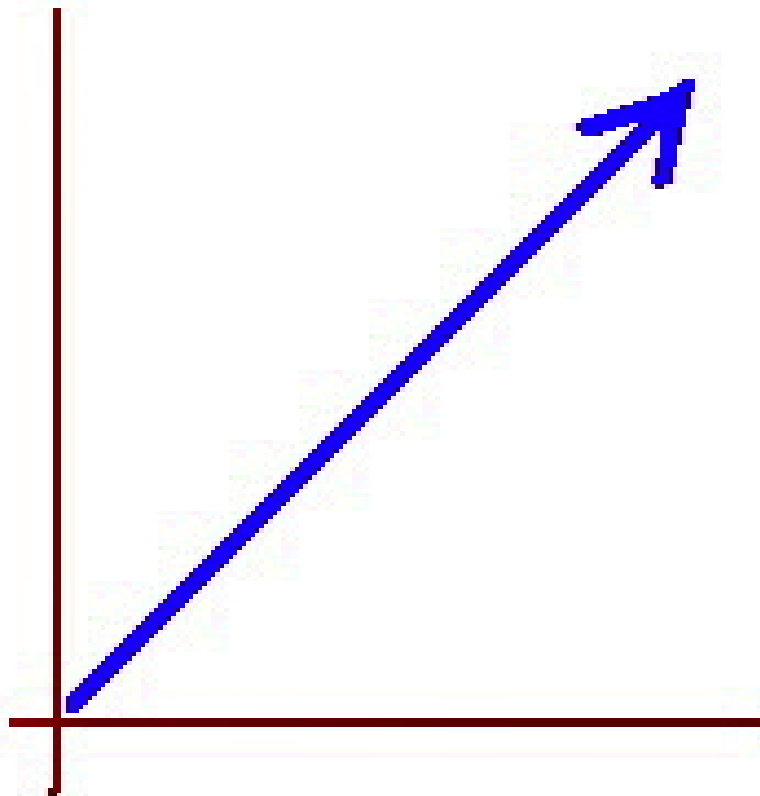


Linear Algebra, Graph Theory, and Finite Geometry

Jaime Guerrero
Dr. Eugene Fiorini

Vectors



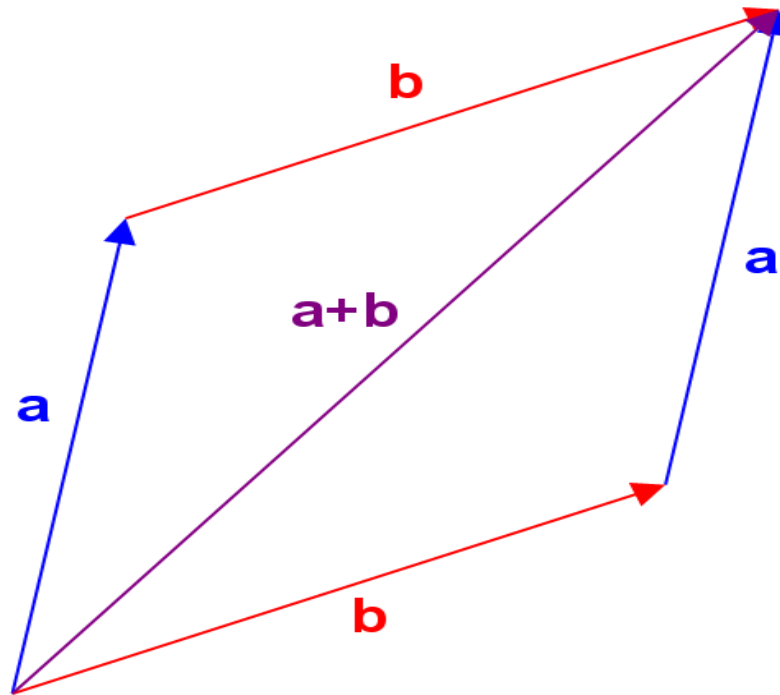
Matrices

Matrices

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -3 & 2 \\ 0 & 0 & 1 & 2 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

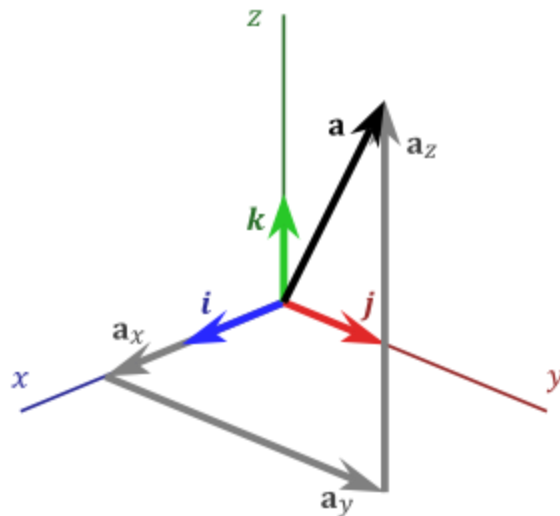
Linear Combinations

- Algebraic operations on vectors
- $a\mathbf{v}+b\mathbf{w}$, where a, b are real numbers



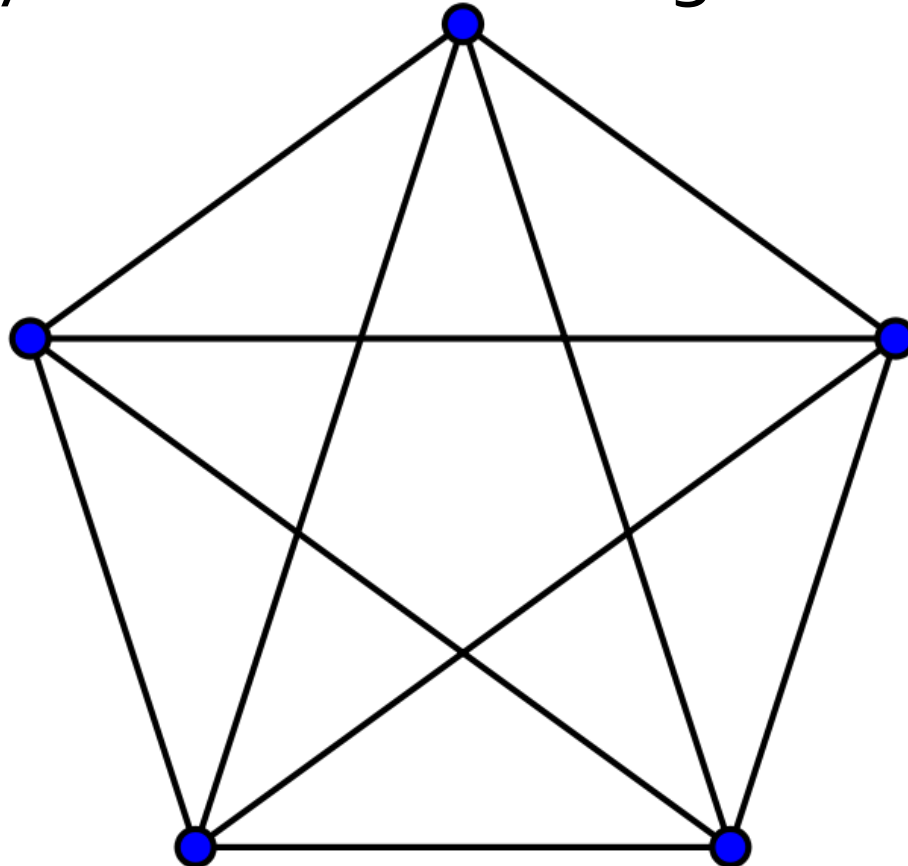
Basis

- Set of vectors
- All vectors in the space are defined through linear combinations of basis vectors
- Provides a definition for *dimension*

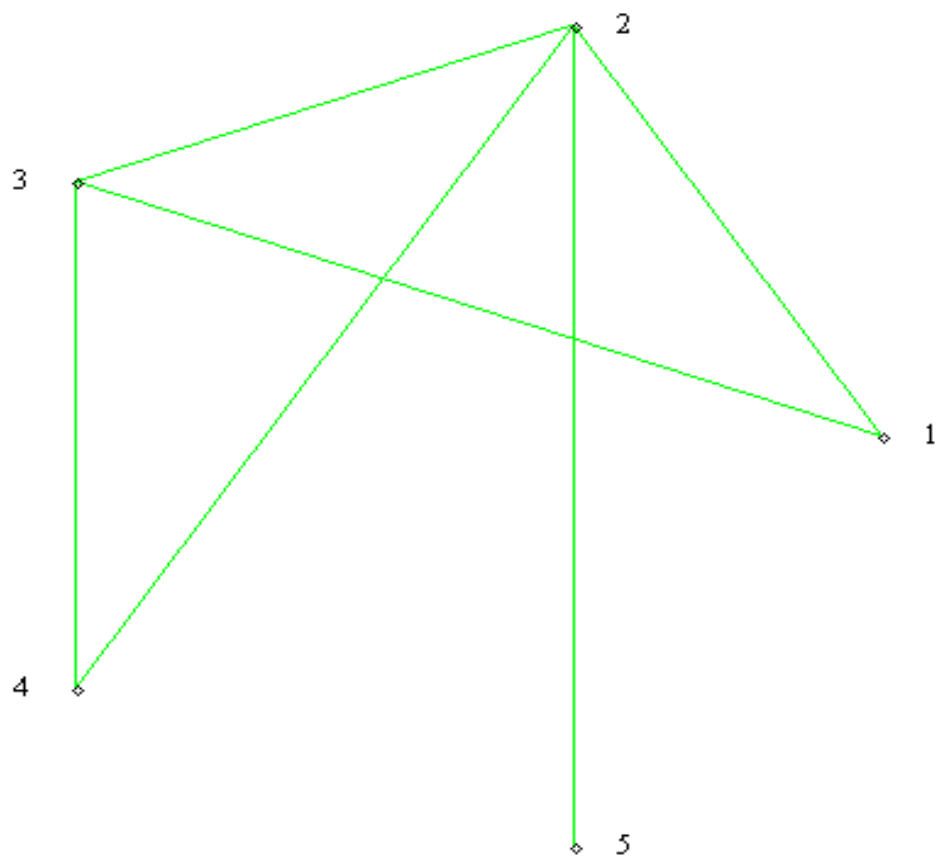


Graph Theory

- The study of vertices and edges

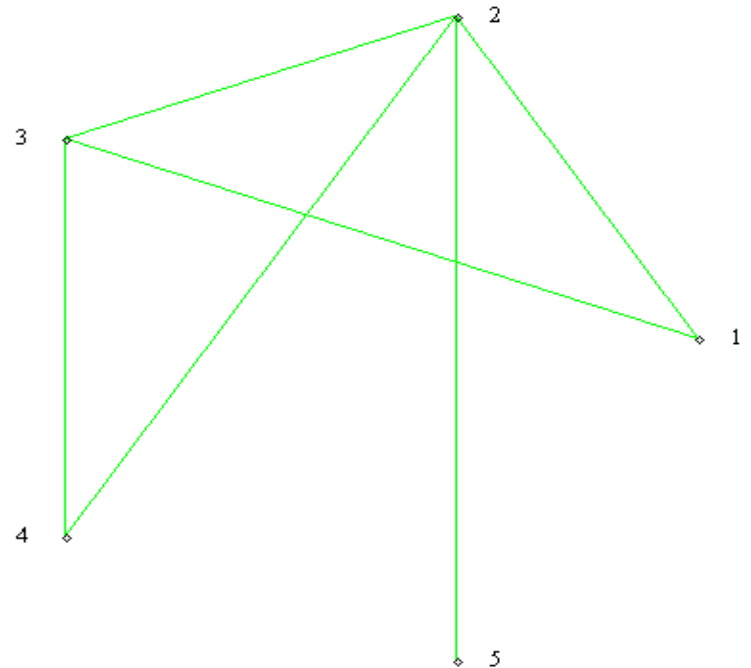


Linear Algebra Connection?



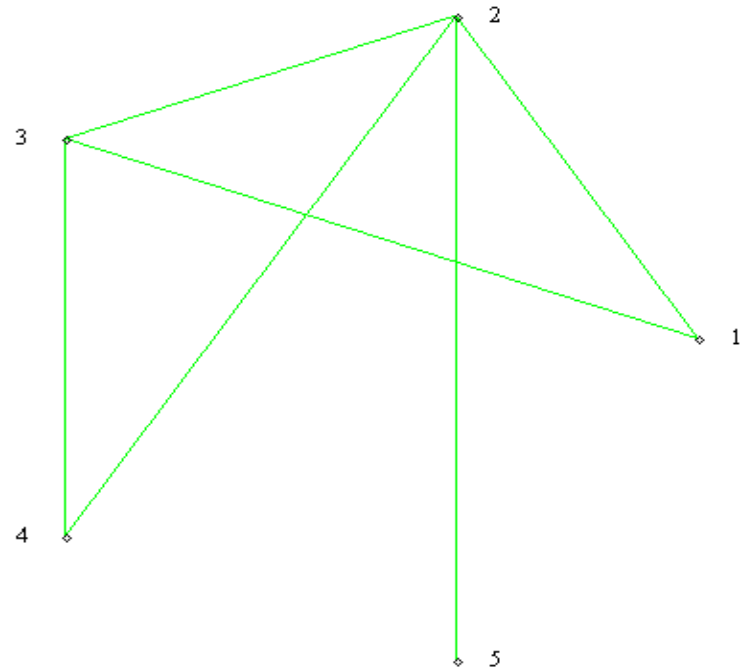
Adjacency Matrix

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$



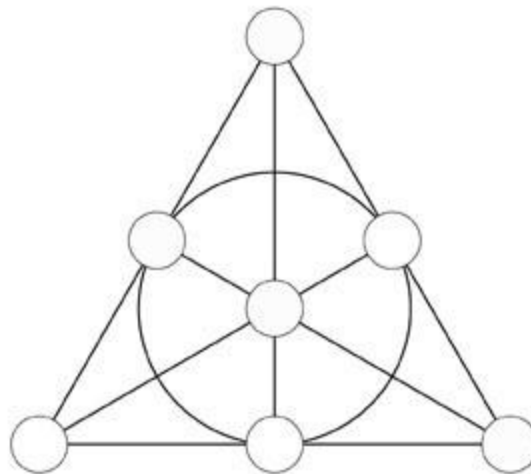
Adjacency, Squared

$$A^2 = \begin{bmatrix} 2 & 1 & 1 & 2 & 1 \\ 1 & 4 & 2 & 1 & 0 \\ 1 & 2 & 3 & 1 & 1 \\ 2 & 1 & 1 & 2 & 1 \\ 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$



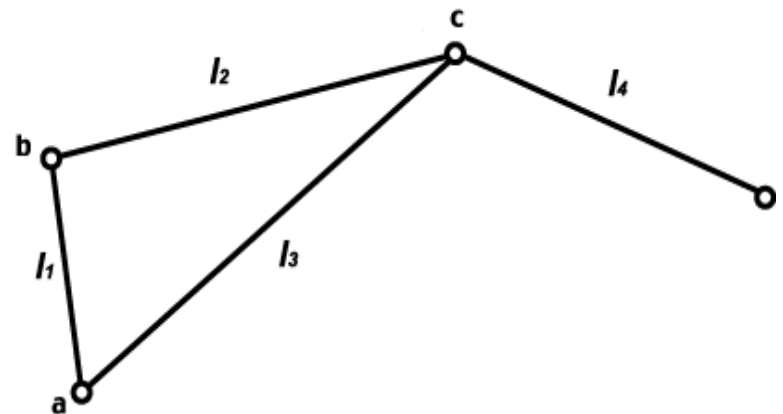
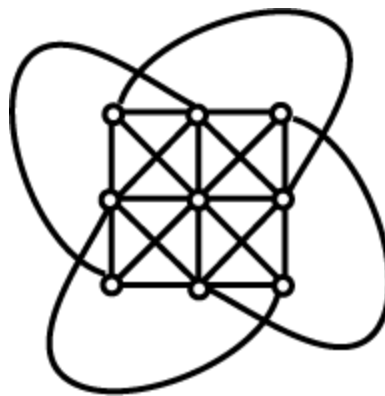
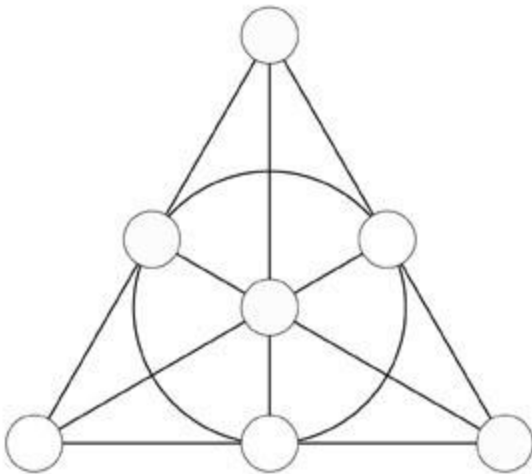
Finite Geometry

- Not what you learned in high school
- Geometry with a finite number of points and lines
- Different axioms
- NOT 2- or 3-dimensional space



Finite Examples

- Near-linear spaces
 - Each line has two or more points
 - Each point is on two or more lines

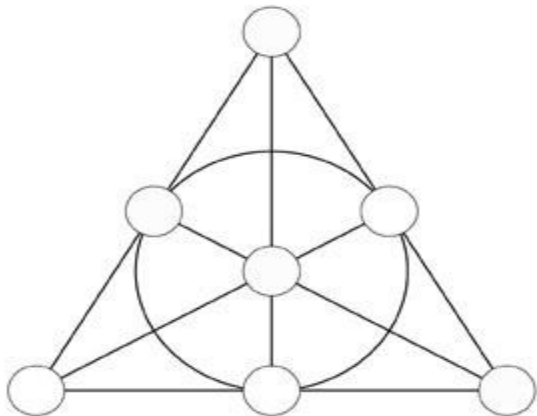


Example

- There are seven points and seven lines
- Every pair of distinct points identifies a unique line
- Every pair of distinct lines intersects at a unique point.

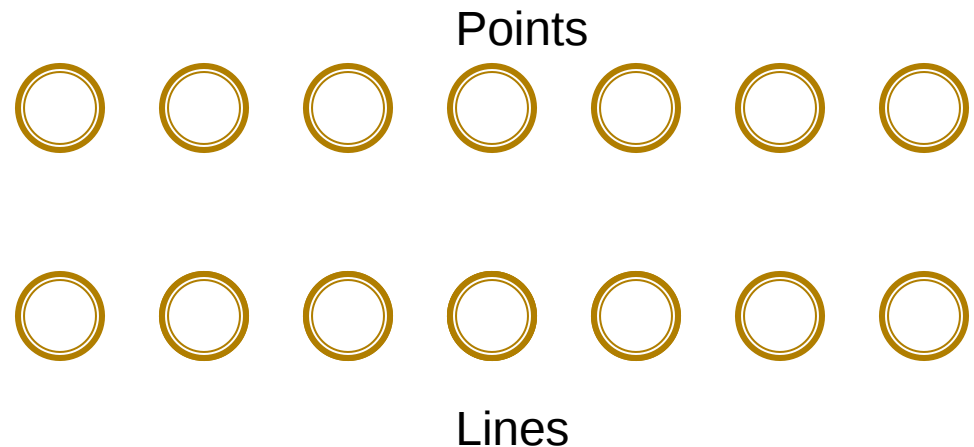
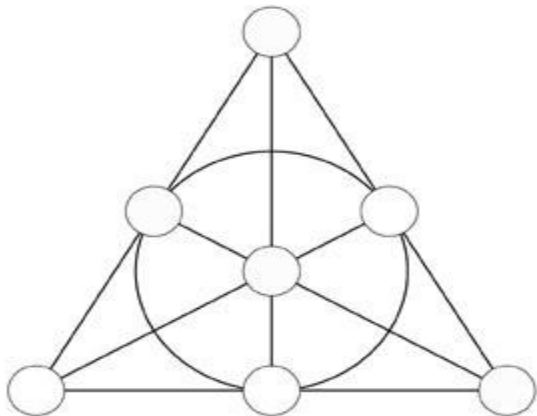
Fano Plane

- There are seven points and seven lines
- Every pair of distinct points identifies a unique line
- Every pair of distinct lines intersects at a unique point.



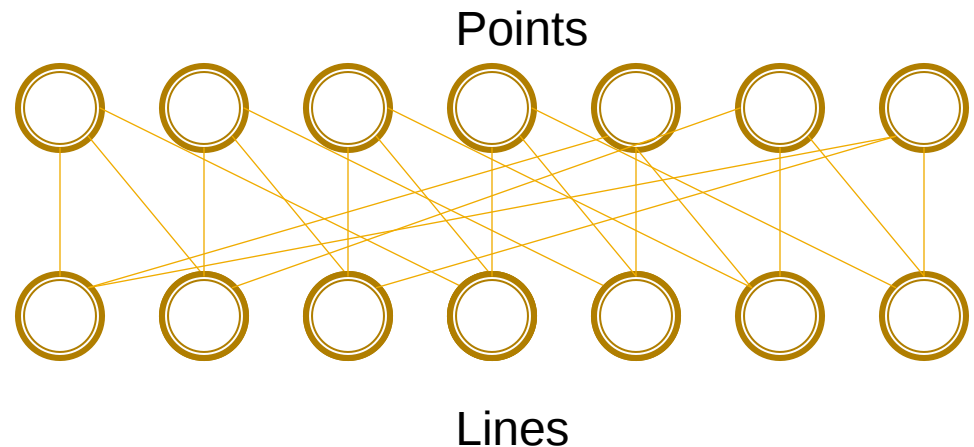
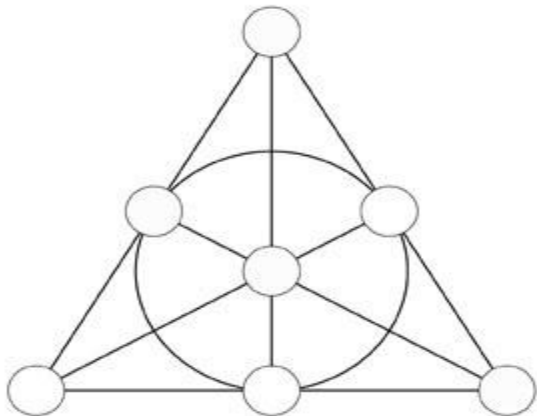
Fano Plane

- There are seven points and seven lines
- Every pair of distinct points identifies a unique line
- Every pair of distinct lines intersects at a unique point.



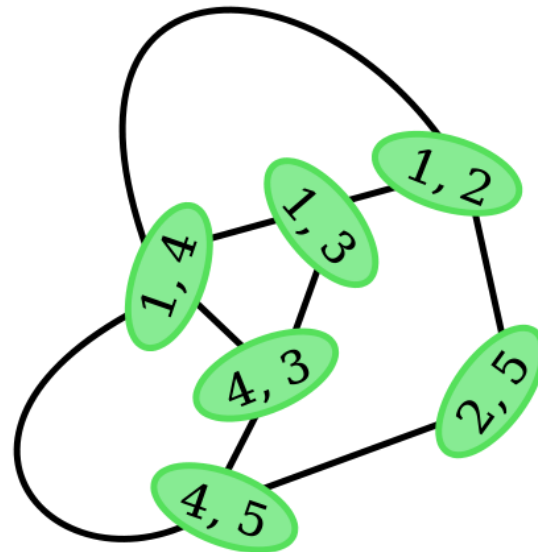
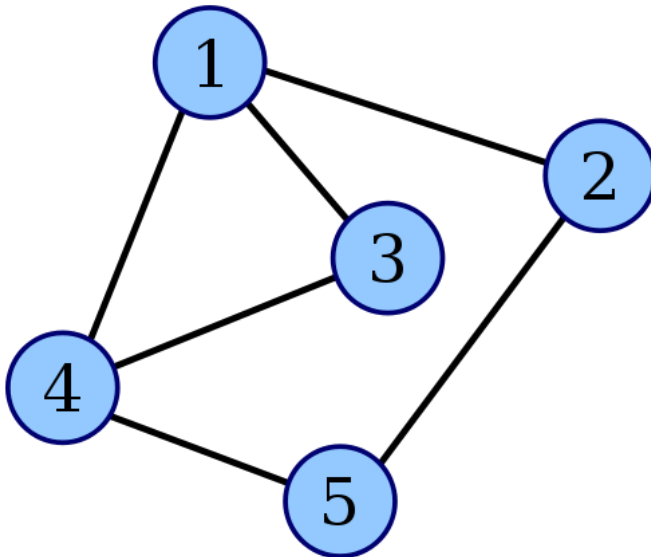
Fano Plane

- There are seven points and seven lines
- Every pair of distinct points identifies a unique line
- Every pair of distinct lines intersects at a unique point.



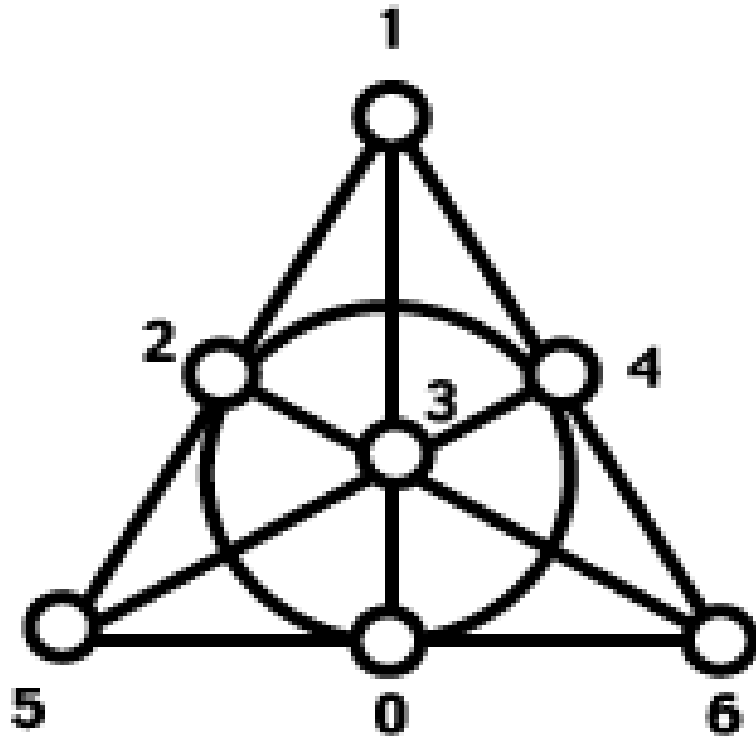
Line Graph

- Given a near-linear space $S=(P,L)$
 - Define $R=(P',L')$
 - $P' = L$
 - $L' = \{\{p,q\}\}$



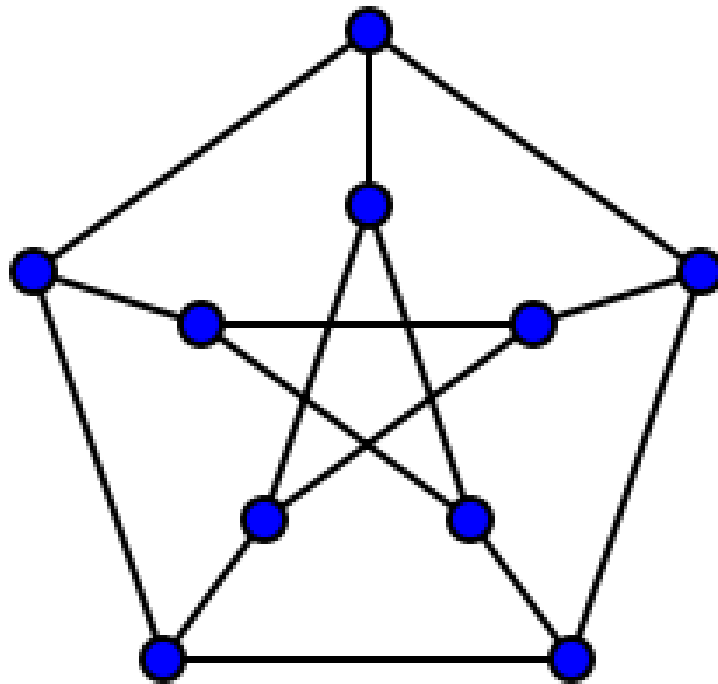
Point Graph

- Points become vertices
- The vertices are connected if 2 lie on one line



Parent Graph

- The “larger” graph that gives you the “smaller” geometry



Research

- What can the adjacency matrix tell us about embedded geometries?
- How is the embedded geometry of a product graph related to its factors?
- What role do eigenvalues play?