

Continuum Limit of Bell's Jump Process in Lattice Quantum Field Theory

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7 June 2011

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My research will focus on an aspect of the relationship between two hidden variable theories, a continuous model called Bohmian Mechanics and a discrete jump process called Bell's Jump Process.

The goal is to show that Bohmian Mechanics is the continuum limit of Bell's Jump Process with suitable choices of Hilbert space and Hamiltonian.

I have been working on this in one dimension for several months, after which the goal will be to generalize the result to N dimensions.

Why look at hidden variables?

The Measurement Problem

Under the quantum formalism, that is the set of rules used to predict the outcome of equations, we are told that the wavefunction evolves **deterministically** according to the Schrödinger Equation.

However, when a measurement occurs, then the wavefunction **randomly** “collapses” into an eigenstate.

A Measurement

“What exactly qualifies some physical systems to play the role of ‘measurer’? Was the wavefunction of the world waiting to jump for thousands of millions of years until a single-celled living creature appeared? Or did it have to wait a little longer, for some better qualified system...with a Ph.D.? If the theory is to apply to anything but highly idealised laboratory operations, are we not obliged to admit that more or less ‘measurement-like’ processes are going on more or less all the time, more or less everywhere?”

-John Bell

Bohmian Mechanics: from 1952

Bohmian Mechanics is a quantum mechanical theory of the motion of point particles.

- The k -th particle has a definite position in physical space, $Q_k(t)$.
- There is a wavefunction $\psi_t(q) = \psi(q, t) : \mathbb{R}^{3N} \times \mathbb{R} \rightarrow \mathbb{C}$.
- The time evolution of the wavefunction is given by the Schrödinger equation:

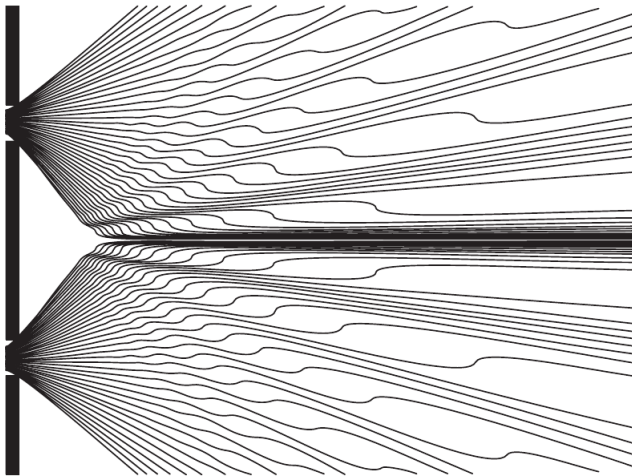
$$i\hbar \frac{\partial \psi}{\partial t}(q, t) = - \sum_{k=1}^N \frac{\hbar^2}{2m_k} \nabla_k^2 \psi(q, t) + V(q) \psi(q, t) = H \psi(q, t) \quad (1)$$

- The possible motion of the particles is governed by the Guiding Equation:

$$\frac{dQ_k}{dt} = \frac{\hbar}{m_k} \operatorname{Im} \left(\frac{\psi^* \nabla_k \psi}{\psi^* \psi} \right) (Q(t), t) = v_k(Q(t), t) \quad (2)$$

The picture to have in mind here is one of particles (with definite positions!) whose motion is guided by the wavefunction.

Some Bohmian Trajectories



This is 80 possible paths of a Bohmian particle through a double slit. While the wavefunction passes through both slits, each particle passes through only one.

A Note on Bohmian Mechanics

It has been proven that statistical treatment of Bohmian Mechanics gives the same predictions as the quantum formalism.

The quantum formalism is a **consequence** of Bohmian Mechanics.

Bell's Jump Process

In 1986, John Bell proposed a stochastic process for lattice quantum field theory that is similar to Bohmian Mechanics.

Let L be a lattice in $\mathbb{R}^3$, for example $L = a\mathbb{Z}^3$, and $\mathcal{Q}$ be a suitable configuration space on L , for example $\mathcal{Q} = L^N$ or $\mathcal{Q} = \bigcup_{N=1}^{\infty} L^N$.

There is a definite configuration $X_t \in \mathcal{Q}$ at each time t .

The picture to have in mind here is one of particles hopping, being created or annihilated so that the state moves between configurations x and y with a given transition rate $\lambda(x \rightarrow y)$.

Bell's Jump Process

Mathematically, we define Bell's Process X_t by:

- There is a state vector $\psi_t \in \mathcal{H}$.
- The state vector evolves according to the Schrödinger Equation as usual:

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi. \quad (3)$$

- There is a countable set $\mathcal{Q}$.
- For every $x \in \mathcal{Q}$ there is a positive operator P_x with $\sum_x P_x = 1$.
(For example $P_x = |x\rangle\langle x|$)
- The process X_t jumps from $x \in \mathcal{Q}$ to $y \in \mathcal{Q}$ in time dt with probability:

$$\lambda(x \rightarrow y)dt = \frac{2}{\hbar} \frac{[\text{Im}(\langle \psi_t | P_x H P_y | \psi_t \rangle)]^+}{\langle \psi_t | P_x | \psi_t \rangle} dt \quad (4)$$

A Quick Comparison

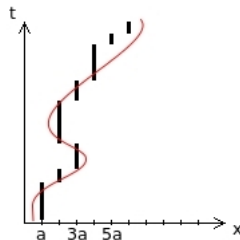
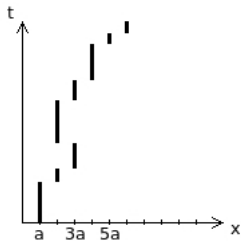
Bohmian Mechanics

- Schrödinger's Equation
- $\frac{dQ_k}{dt} = \frac{\text{Probability Current}}{\text{Probability Density}}$

Bell's Jump Process

- Schrödinger's Equation
- $\lambda(x \rightarrow y) = \frac{\text{Probability Current}}{\text{Probability Density}}$

Possible Trajectories



Our conjecture is that Bell's Process, X_t , converges to a Bohmian trajectory Q_t in the continuum limit.

Thank you for your attention

- [1] D. Dürr, S. Teufel: Bohmian Mechanics, Springer-Verlag, Heidekberg, 2009. The Physics and Mathematics of Quantum Theory
- [2] J.S. Bell: Beables for quantum field theory. Phys. Rep. 137: 49-54 (1986). Reprinted in: Bell, J. S.: Speakable and Unspeakable in Quantum Mechanics, 2nd edition, Cambridge University Press (2004) p. 173-180