Reu Problems of the Czech Group

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Complexity of Fair Deletion Problems

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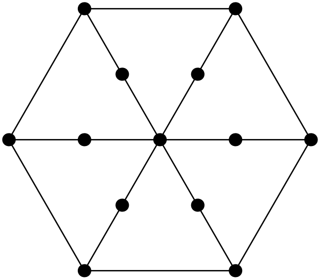
However, they are fixed-parameter tractable (in short FPT)

Fair Versions of Deletion Problems

Sometimes the smallest set is not what we want

- We have a communication network, we want to remove redundancies
- We don't want to reduce the capability of each vertex by too much (a local condition in contrast to the previous global one)



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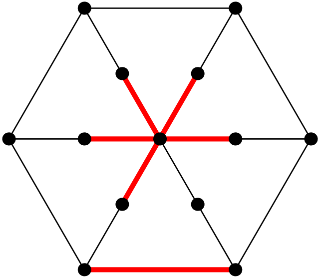

This solution removes a lot of links from single vertex

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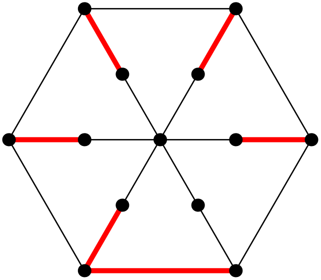

A much better solution

Our Aim

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- There is a result stating that every problem definable in a modification of MSO logic can be solved by an FPT algorithm (Kolman et al.)
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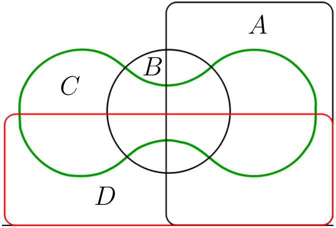
- We plan to tackle this problem by extending the so-called homomorphism-based method by Borie et al.
- The current parameterization is by tree-width; we want to explore different parameterizations that will lead to FPT algorithms

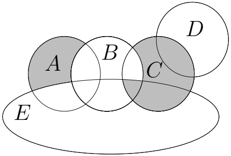
Simplification of the Inclusion-exclusion Formula

Inclusion-exclusion Principle



$$
\begin{aligned}
|A \cup B \cup C \cup D| = & \\
& |A| + |B| + |C| + |D| \\
& -|A \cap B| - |A \cap C| - |A \cap D| - |B \cap C| - |B \cap D| - |C \cap D| \\
& +|A \cap B \cap C| + |A \cap B \cap D| + |A \cap C \cap D| + |B \cap C \cap D| \\
& -|A \cap B \cap C \cap D|
\end{aligned}
$$

Simplified Formula



$$|A \cup B \cup C \cup D| = |A| + |B| + |C| + |D| - 3|A \cap B \cap C \cap D|$$

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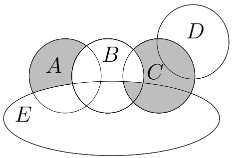

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Short formulas can be found by methods from linear algebra.

Simplified Formula 2

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Problem: In practice, evaluating formulae with large coefficients amplify measurement errors.

We want the formula to be

- short,
- with small coefficients,
- easy to find.

Research Problems

Considering formulas with coefficients from the set $\{-1, 1, 0\}$.

Q: Given m nonempty regions what is the shortest formula?

Q: Is it NP-hard to decide whether there is a formula with at most k terms?

Known results:

Lower Bound: There are families of sets requiring $\Omega(m^{2-\varepsilon})$ terms.

Upper Bound: There is an algorithm which finds a formula of size $m^{\mathcal{O}(\log^2(n))}$ in expected running time $m^{\mathcal{O}(\log^2(n))}$.

Stronger results for specific families of sets (e.g. disks in a plane).

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We will focus on

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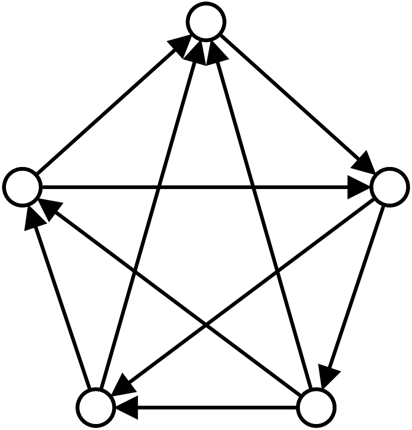
- Complexity of the decision problem about existence of a short formula with $\{-1, 0, 1\}$ coefficients.
- Studying structural properties of specific set systems described by Venn diagram regions.
- Generalization of the results for specific families of set systems.
- Improving the upper bound.

Dominating Sets on Colored Tournaments

Dominating Set on Tournaments

- A tournament is a complete graph with orientated edges.
- A vertex v is dominated by u if there is an edge $u \to v$.
- A dominating set is a set of vertices called dominators such that every vertex of the graph is a dominator or is dominated by at least one.



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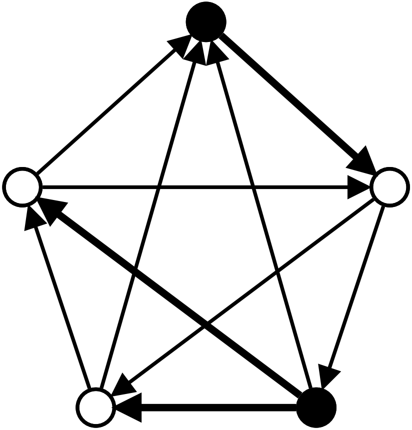

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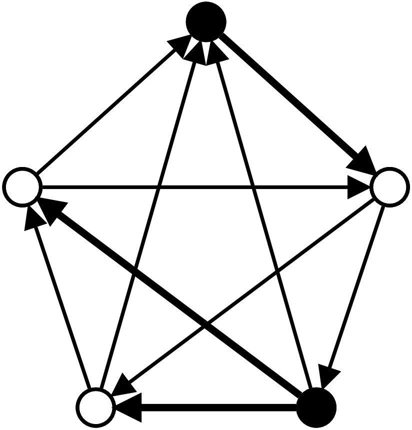
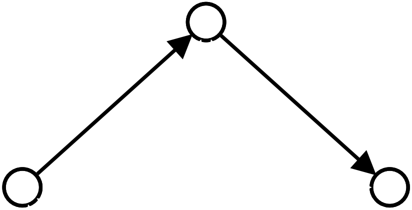


Our goal

We are looking for an upper bound of the size of the dominating set on colored tournaments.

Edge-transitive Tournaments

- An edge-transitive digraph: $u \to v \& v \to w \implies u \to w$
- This means the graph must be acyclic.
- On transitive tournaments, one vertex dominates all.



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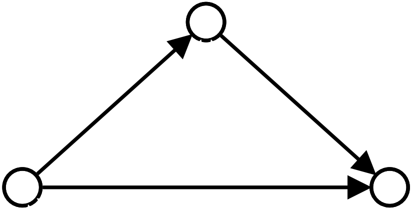

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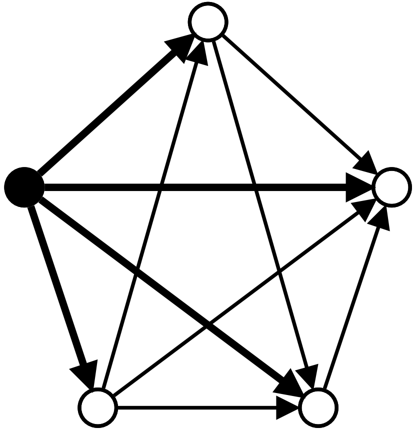
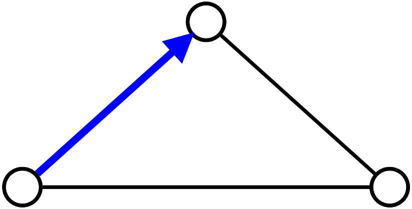


2-colored Transitive Tournaments

- Every edge is either red, or blue.
- Edges of the same color satisfy transitivity.
- Then whole graph is transitive and it has a single dominator.



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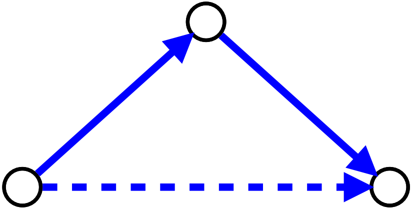

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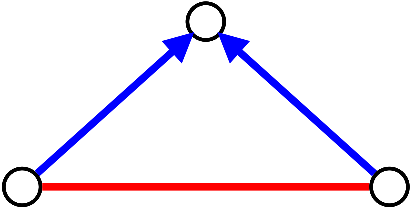

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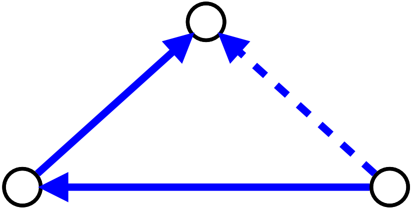

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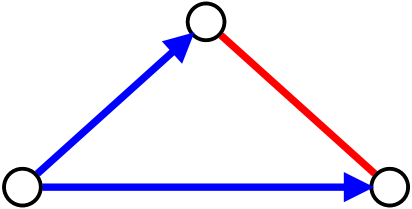

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Plan

We will test small cases algorithmically and then try to generalize the result for tournaments of an arbitrary size.

Subclasses of Chordal Graphs

Symmetries of Graphs

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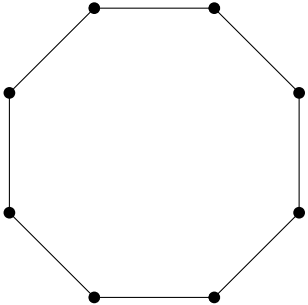

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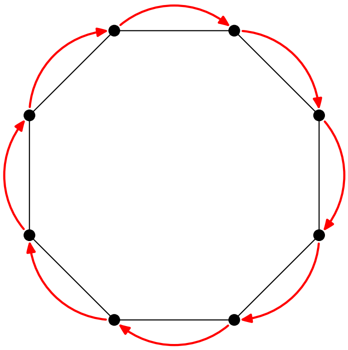

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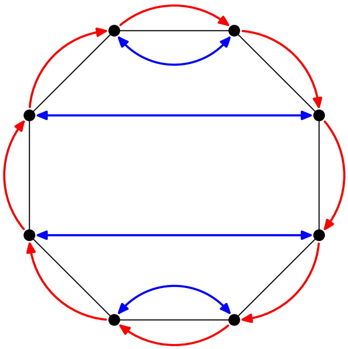
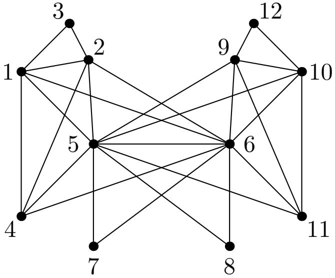
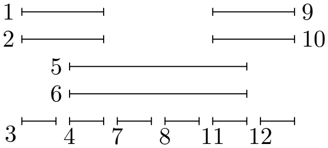


Interval Graphs (INT)

Interval graphs are intersection graphs of intervals of the real line.





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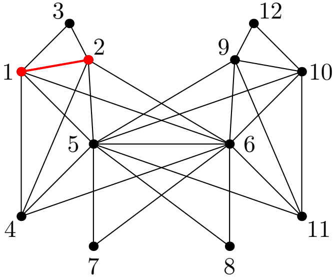

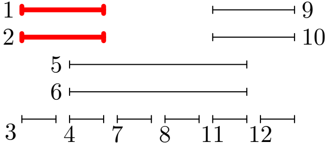

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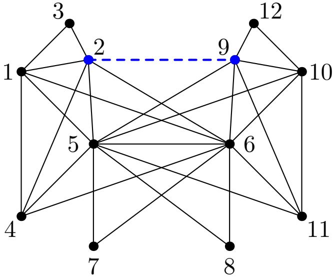

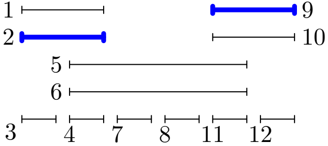

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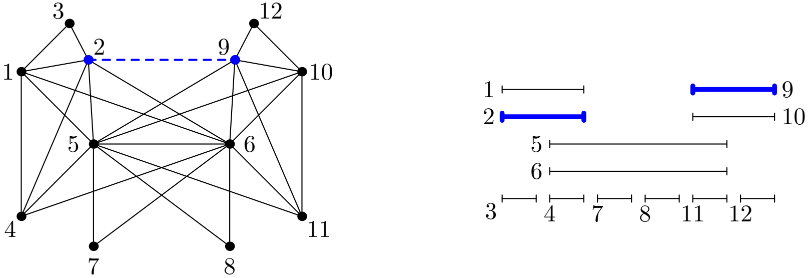

Many computational problems are polynomially solvable for interval graphs (graph isomorphism, maximum clique, *k*-coloring, maximum independent set, ...).

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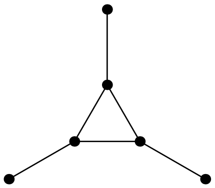

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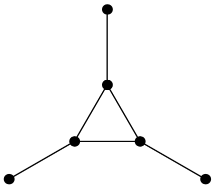

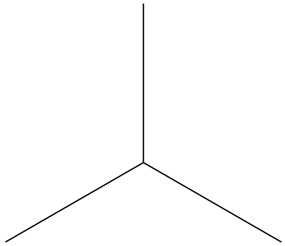

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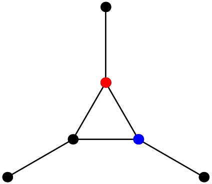

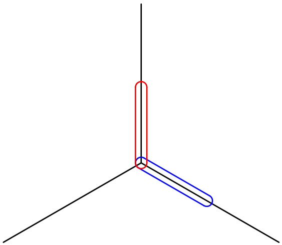

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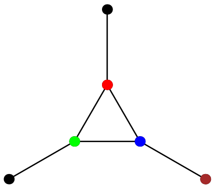

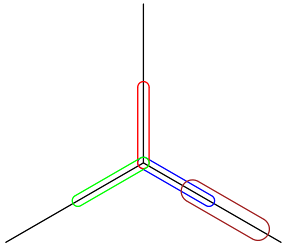

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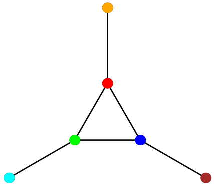

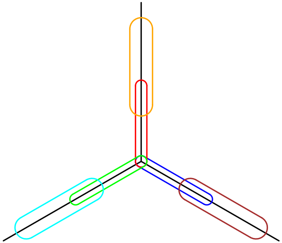

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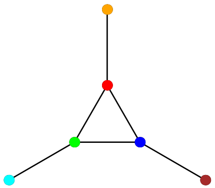

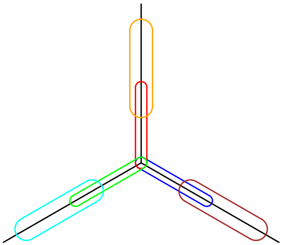

It is easy to see that INT $\subsetneq$ CHOR.

T-GRAPH

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We obtain an infinite hierarchy between INT and CHOR, since

$$\text{INT} \subseteq T\text{-GRAPH} \subsetneq \text{CHOR}.$$

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We obtain an infinite hierarchy between INT and CHOR, since

$$\text{INT} \subseteq T\text{-GRAPH} \subsetneq \text{CHOR}.$$

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Concerning the class T-GRAPH, not much is known in comparison to INT and CHOR.

Main Aim

Our main aim is to study the structural and computational properties of the classes T-GRAPH.

First, we will try to understand the classes T-GRAPH, for some special tree T (for example $K_{1,n}$).

It is possible that some techniques developed for interval graphs could be used.

Thank you!