Computational complexity of the packing coloring problem
(suggested by Tomáš Masařík)

**Definition** (Packing $k$-coloring [2]). *Given a graph $G = (V, E)$ and integer $k$, a* packing $k$-coloring *is a mapping $c : V \to \{1, \ldots, k\}$ such that any two vertices $u, v$ of color $c(u) = c(v)$ are in distance at least $c(u)$ $(d(u, v) > c(u))$.*

| | |
|---|---|
| PACKING $k$-COLORING OF A GRAPH | |
| **Input:** | A graph $G$ a positive integer $k$. |
| **Question:** | Does graph $G$ allow a packing $k$-coloring? |

**Previous results**

- Goddard et al [2] showed that the PACKING COLORING problem is NP-complete for general graph and $k = 4$ and it is polynomial time solvable for $k \leq 3$.
- Fiala et al [1] showed that the PACKING COLORING problem is NP-complete for trees for large $k$ (dependent on the number of vertices). For fixed $k$ the problem is expressible in MSO logic and thus due to Courcelle's theorem it admits an FPT algorithm on graphs of bounded treewidth. Moreover, it is solvable in polynomial time for graphs of bounded both the treewidth and the diameter [1].
- The problem is NP-complete for chordal graphs of diameter at most 5 [1] and it is polynomial for chordal graphs of diameter at most 3 [2]. However it admits an FPT algorithm on chordal graphs parameterized by $k$ [1].

**Questions**

- What is a complexity of the PACKING COLORING problem on chordal graphs of diameter 4?
- What is a complexity of the PACKING COLORING problem on (unit) interval graphs (since they are subclass of chordal graphs)?
- What is a complexity of the PACKING COLORING problem on (outer)planar graphs and fixed $k$ (not as a part of the input)?

The plan to solve those questions is to modify existing hardness reductions or at least to get inspired by the problems they are reducing from. On the positive side we should try to derive algorithms for some restricted graph class or possibly we could find a suitable parametrization (number of colors or structural parameters) and then either deduce an FPT algorithm or derive W[t]-hardness. Another option is to get partial results on subclasses (i.e. unit interval graphs or outer-planar graphs) or to step aside and investigate similar graph classes (i.e. permutation graphs).

**Bibliography**

[1] Jiří Fiala and Petr A Golovach. Complexity of the packing coloring problem for trees. *Discrete Applied Mathematics*, 158(7):771–778, 2010.

[2] Wayne Goddard, Sandra M Hedetniemi, Stephen T Hedetniemi, John M Harris, and Douglas F Rall. Broadcast chromatic numbers of graphs. *Ars Combinatoria*, 86:33–50, 2008.