

Kuramoto Model of Synchronization in Complex Multilayer Networks

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Synchronization



Figure 1: Murmuration of Starlings.

<https://www.youtube.com/watch?v=eakKfY5aHmY>

"Starling," *Wikipedia*. Web. Accessed 2 June 2017.

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Synchronization

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What is synchronization?

- Starlings flying
- Crickets chirping
- An audience clapping

Definition: Synchronization

N independent, initially heterogeneously rotating oscillators become *synchronized* when they achieve sufficiently simultaneous phased behavior.

Kuramoto Model

- N number of oscillators
- ω_i natural frequency
- θ_i angle of oscillator i at time t
- λ coupling strength
- r average position of oscillators in complex plane
- ϕ average angle of oscillators

The Model

$$\frac{d\theta_i}{dt} = \omega_i + \frac{\lambda}{N} \sum_{j=1}^N \sin(\theta_j - \theta_i)$$

↓

$$\frac{d\theta_i}{dt} = \omega_i + \lambda r \sin(\phi - \theta_i)$$

Kuramoto Model

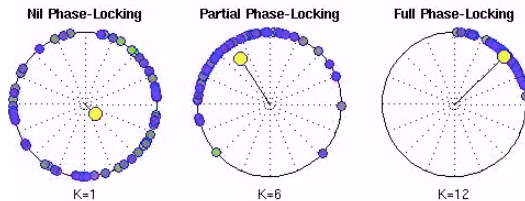


Figure 2: Visualization of Phase Locking for Different Coupling Strengths

"Kuramoto model," *Wikipedia*. Web. Accessed 2 June 2017.

Complex Networks

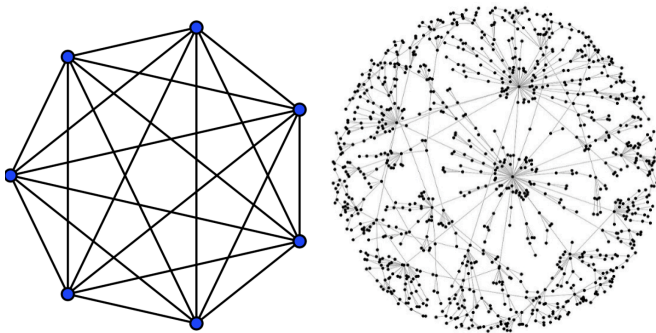


Figure 3: Complete Network vs. Sparse Network

"Complete graph," *Wikipedia*. Web. Accessed 4 June 2017.

Cockell, Simon. "Sparse, scale-free network." Flickr, 29 Jan. 2013. Web. Accessed 4 June 2017.

Questions

- 1 How does a network's topology affect synchronization?
- 2 How does a multilayer structure affect synchronization in each layer?

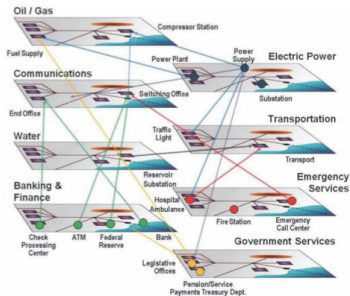


Figure 4: Multilayer Network (Fefferman et al)

Conclusion

Thank you for listening!

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- ▶ Rodrigues, F. et al. “The Kuramoto model in complex networks.” *Physics Reports* 610 pp. 1-98, Oct. 2015. Elsevier.
- ▶ Acebr on, J. et al. “The Kuramoto model: A simple paradigm for synchronization phenomena.” *Reviews of Modern Physics* 77, pp. 137-185, Jan. 2005. The American Physical Society.