

LINEAR PROGRAMMING

Bridge Workshop Exercises

EXERCISE ONE We are given a list of points (x_i, y_i) on a plane, each of them is the center of a square s_i with sides parallel to main axes. Make a linear program which finds side lengths of all squares, such that the sum of the side lengths is largest possible and the square insides do not intersect each other.

Try finding some tricks to decrease the number of constraints in the resulting linear program (Worst case, it doesn't have to help, but in an "usual" case the linear program will have much less constraints).

EXERCISE TWO Formulate the knapsack problem using integer programming. As in, for n objects, where each of them has mass m_i and a price p_i , we have a knapsack load capacity of H . Our goal is to put the objects into the knapsack in such a way we maximize the sum of their prices.

EXERCISE THREE Construct the dual linear program to this:

$$\begin{aligned} \max x_1 - 2x_2 + 3x_4 \\ x_2 - 6x_3 + x_4 &\leq 4 \\ -x_1 + 3x_2 - 3x_3 &= 0 \\ 6x_1 - 2x_2 + 2x_3 - 4x_4 &\geq 5 \\ x_2 &\leq 0 \\ x_1, x_3, x_4 &\geq 0 \end{aligned}$$

Try coming up with a way to dualize equalities.

EXERCISE FOUR Prove that integer programming is NP-hard.

EXERCISE FIVE As you sure know, sudoku can be solved with a trivial program which just goes through all possible solutions. Can you write sudoku solver as an integer program?