Entire List Coloring

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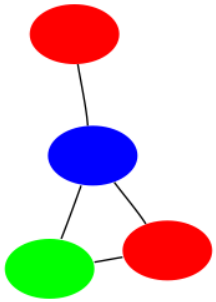
Graph Coloring, A Review



A *Graph* consists of a series of vertices connected by edges.

To *color* a graph means to assign a color to every vertex such that no two vertices that share an edge also share a color.

Many applications of graph coloring call for finding the smallest number of colors necessary to color the graph. This is the *chromatic number*, and is denoted χ.

Entire Coloring

An *Entire Coloring* is much like a vertex coloring, except you also color all the edges and faces of the graph, so that no two adjacent elements share a color. This is denoted χ_{vef}.

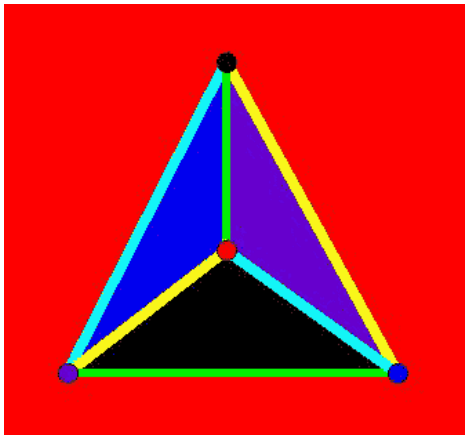


Figure: Example of entire coloring. Image courtesy of `www.ams.org`

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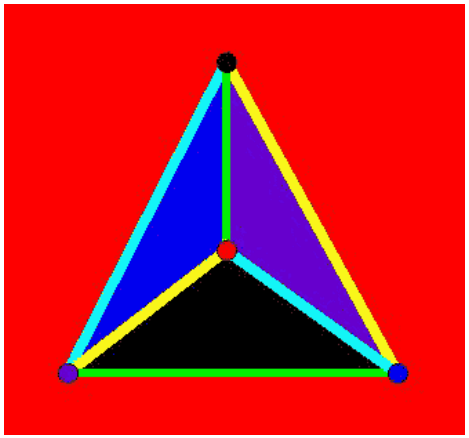

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For all graphs G, $\chi_{vef}(G) > \chi(G)$.

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We wanted to see if we could translate arguments about entire colorings into entire list colorings, and improve on the second lower bound.

Example: 6-List Colorability

Theorem

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Lemma

All planar graphs have a vertex of degree 5 or less.

Proof follows from Euler's formula for planar graphs.

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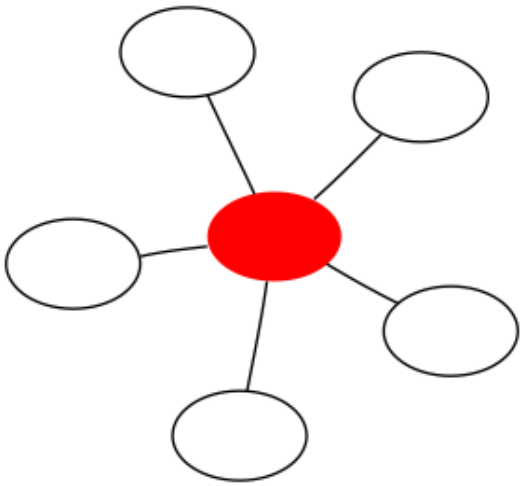
Find this 5^- vertex, and remove it from the graph.



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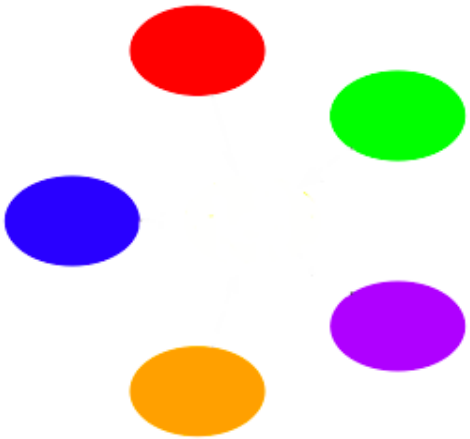

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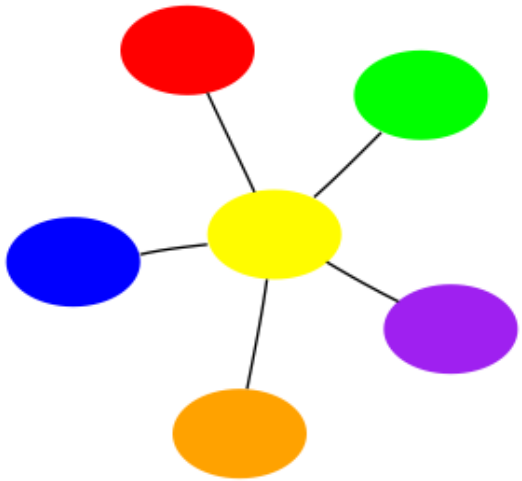

By induction, color all remaining vertices from their lists.

Add the removed vertex back and color it from its list.

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Questions?