1 Some 3x3 versions of Lemma 1

As long as $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ and $\begin{bmatrix} d \\ e \\ f \end{bmatrix}$ are linerly independant, this equation has 0 as an critical-vector of multiplicity 3, a critical-space of dimention 1, a 2 by 2 non-diagonalizable block is possible, and a 3 by 3 non-diagonalizable block is not.

$$X^2 + \begin{bmatrix} a & -1 & * \\ b & 0 & * \\ c & 0 & * \end{bmatrix} X + \begin{bmatrix} 0 & -a & d \\ 0 & -b & e \\ 0 & -c & f \end{bmatrix} = 0$$

Same thing with mulitplicity 4 ($b \neq e$)

$$X^2 + \begin{bmatrix} a & -1 & * \\ b & 0 & * \\ c & 0 & * \end{bmatrix} X + \begin{bmatrix} 0 & -a & a \\ 0 & -b & e \\ 0 & -c & c \end{bmatrix} = 0$$

Same thing with mulitplicity 5 ($b \neq e$)

$$X^2 + \begin{bmatrix} a & -1 & a+1 \\ b & 0 & * \\ c & 0 & c \end{bmatrix} X + \begin{bmatrix} 0 & -a & a \\ 0 & -b & e \\ 0 & -c & c \end{bmatrix} = 0$$

Same thing with mulitplicity 6 ($b \neq e$)

$$X^2 + \begin{bmatrix} a & -1 & a=1 \\ b & 0 & * \\ -a & 0 & -a \end{bmatrix} X + \begin{bmatrix} 0 & -a & a \\ 0 & -b & e \\ 0 & a & -a \end{bmatrix} = 0$$

2 Other cool stuff

One of the critical spaces of this equation is two dimentional. However, this equation still has finitley many (one) solutions.

$$X^2 + \begin{bmatrix} 0 & 1 & 1 \\ 0 & -3 & -4 \\ 0 & 1 & 1 \end{bmatrix} X + \begin{bmatrix} 0 & -1 & -1 \\ 0 & 2 & 4 \\ 0 & -1 & -2 \end{bmatrix} = 0$$

This equation has exactly 11 solutions

$$X^2 + \begin{bmatrix} 2 & -1 & 0 \\ \frac{8}{3} & -2 & 0 \\ 3 & -3 & 0 \end{bmatrix} X + \begin{bmatrix} 0 & -2 & 0 \\ 0 & -\frac{8}{3} & 0 \\ 0 & -3 & -1 \end{bmatrix} = 0$$

This equation has 6 distinct critical vectors. If you look at a particular 4 of them, no set of three of those vectors are linearly independant. I believe 4 is the maximum size of such a set.

$$X^2 + \begin{bmatrix} .6 & -1.2 & -1.4\bar{6} \\ -1.2 & -.6 & -1.0\bar{6} \\ 0 & 0 & -3 \end{bmatrix} X + \begin{bmatrix} -1.6 & -1.2 & 0 \\ 1.2 & 1.6 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$