A Game Theory Approach to Cascading Behavior in Networks

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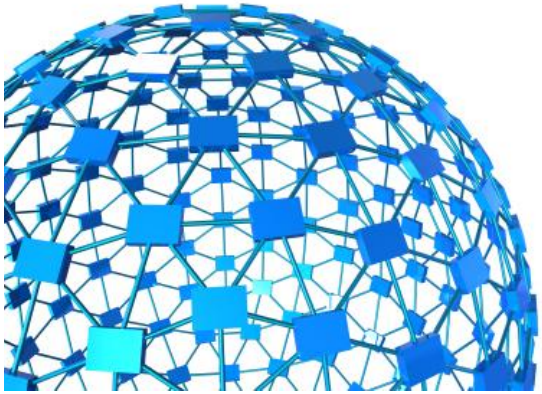
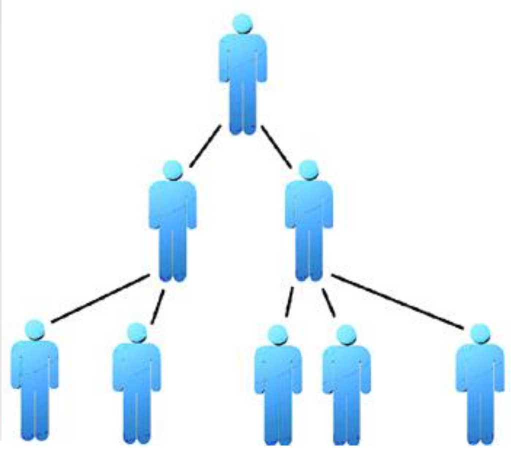


What is Viral Marketing?

- Start off with a small group of individuals
- "Word-of-Mouth"
- Initial individual tells two of his friends and they tell…
- Not necessarily selling a product
- Viral ➔ Think Virus



- Two people start singing
- Next their friends join in
- By the end of the song, the whole bar is singing Mariah Carey's Fantasy…true story

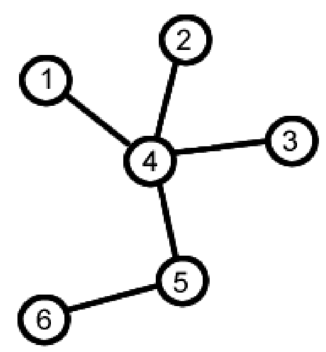
Small-Scale Behavioral Viral Spreading

Graph Theory Basics

Nodes

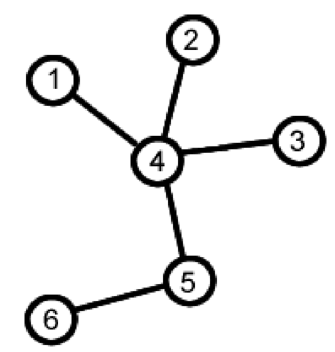
Edges

Directed & Undirected



Graph Theory relating back to VM

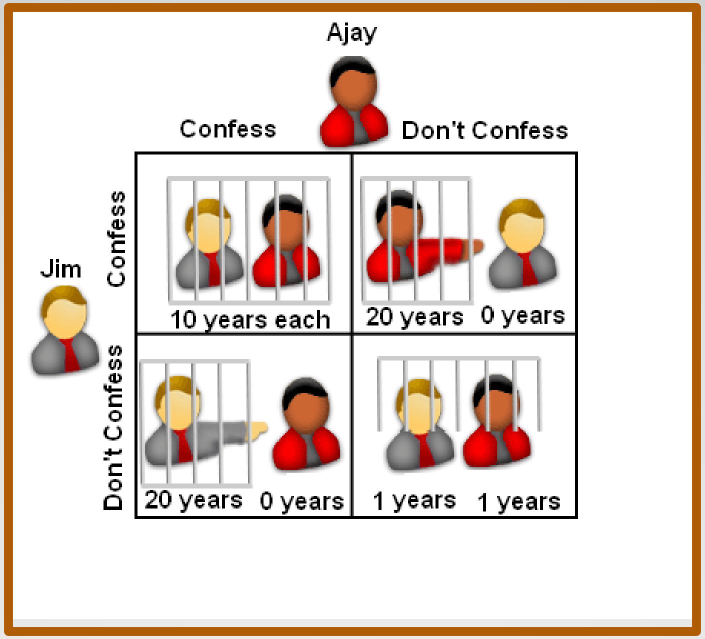
- Contagious
- Progressive & Nonprogressive
- Weighted edges
- “Target” node



Game Theory

- Prisoners' Dilemma
- Two choices
- Confess
- Don't Confess

- Payoff
- Prisoners' Outcomes
- Payoff Table



- Zero Sum and Nonconstant Sum Games
- Nash Equilibrium

	Confess	**Don't Confess**
Confess	(10,10)	(0,20)
Don't Confess	(20,0)	(1,1)

- Clean Kitchen: $50, Clean Basement: $30, Clean Living Room: $20

	Kitchen	Basement	Living Room
Kitchen	(25,25)	(50,30)	(50,20)
Basement	(30,50)	(15,15)	(30,20)
Living Room	(20,50)	(20,30)	(10,10)

- Apply Game Theory to Viral Marketing
- Who are the players?
- Behaviors

Agent i's Payoff

- $v_i\ g(d_i)\ \pi_i - c_i$
  - v_i = benefit from switching
  - $g(d_i)$ = how number of neighbors affects switch
  - π_i = % of neighbors already switched
  - c_i = cost of switching
- Will switch if $(v_i / c_i)\ g(d_i)\ \pi_i > 1$
- Let F be cdf of v_i / c_i

Number of Neighbors Effect

- One possible $g(d) = \alpha d^{\beta}$

- If $\beta=0$: Only fraction of neighbors activated matters. Matched games.
- If $\beta=1$: $g(d_i)\ \pi_i$ is proportional to number of neighbors activated. Epidemiology.

Determining Percent Activated

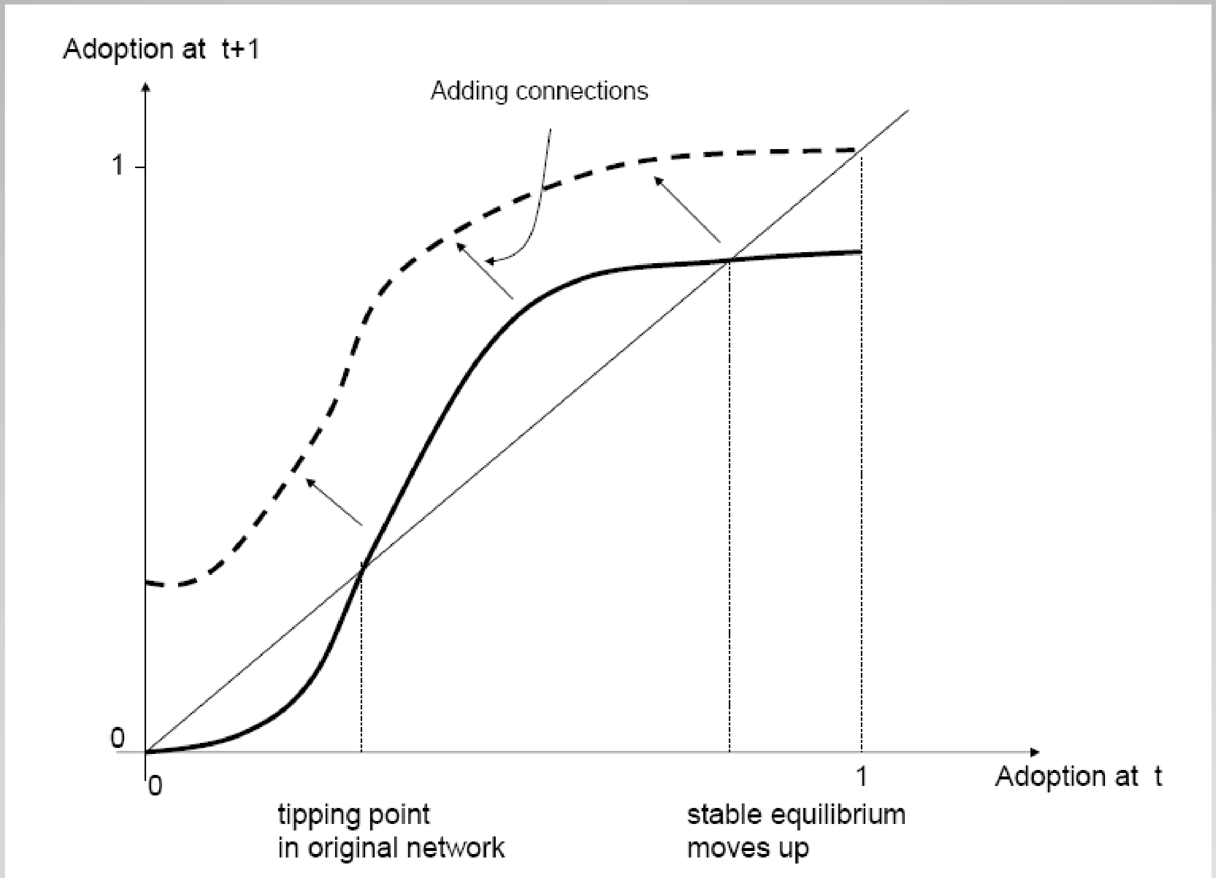
- P(d) is the connectivity distribution
  - i.e. percentage of agents in population with exactly d direct neighbors
- x^t = percentage activated at time t

$$x^t = 1 - \frac{1}{\bar{d}} \sum_d d\, P(d) F\left[\frac{1}{g(d) x^{t-1}}\right]$$

Connectivity Distribution

- Three example types:
  - Scale-free networks (power law)
    - $P(k) \propto k^{-\gamma}, \qquad 2 < \gamma < 3$
  - Homogenous Networks
    - $P(k) \sim \begin{cases} 0, & k \neq \bar{k} \\ 1, & k = \bar{k} \end{cases}$
  - Poisson Networks
    - $P(k) = \dfrac{1}{k!} e^{-\bar{k}} \bar{k}^{k}$
- Larger variance => Product spreads better

Tipping Point and Equilibrium



Most Influential Nodes

- In general, identifying k most influential nodes is NP-hard.
- A natural greedy algorithm exists which is a $1-1/e-\varepsilon$ approximation for selecting a target set of size k, using probabilistically determined simulations.

Our Research Direction

- Looking for graph types which are not susceptible to strong inapproximability results, and for which good approximation results can be obtained.
- Consider graphs with edges weighted differently.
- Analyze real world data from social networks or viral marketing campaigns.

Sources

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Thank you for your attention!

Questions?