Graph Pebbling

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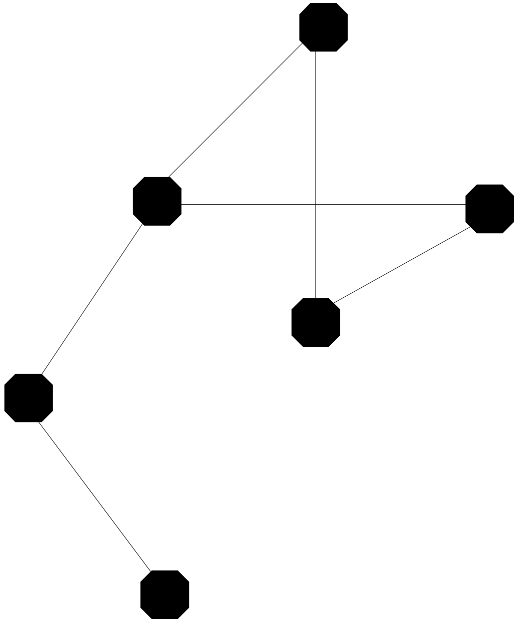
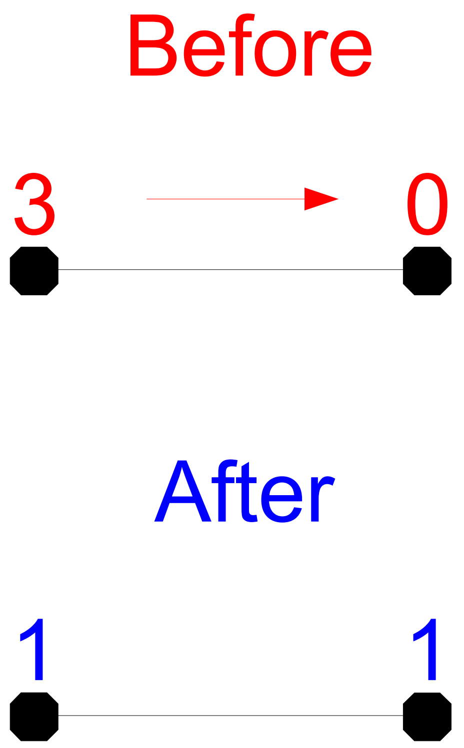
Graph Overview

- A graph *G* is a set of vertices *V(G)* together with a set of edges *E(G)* connecting the vertices
- Concerned with simple, connected graphs



Pebbling Move

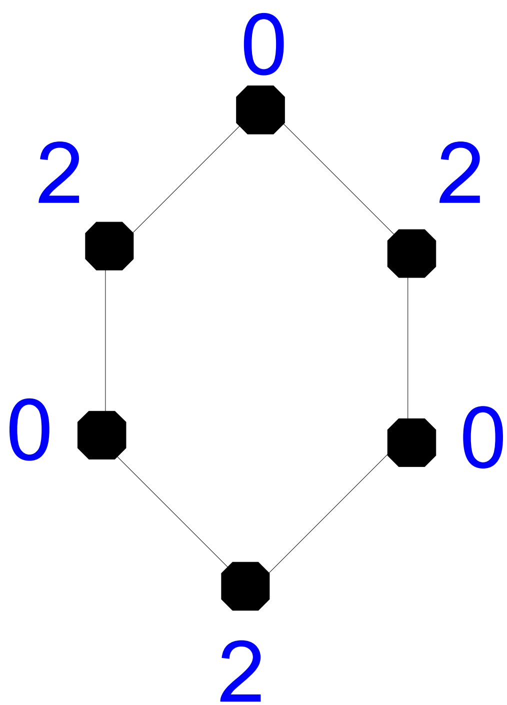
- A configuration is a distribution of pebbles onto *V(G)*
- Move pebbles around by removing two pebbles at a vertex and placing one pebble at a neighboring vertex



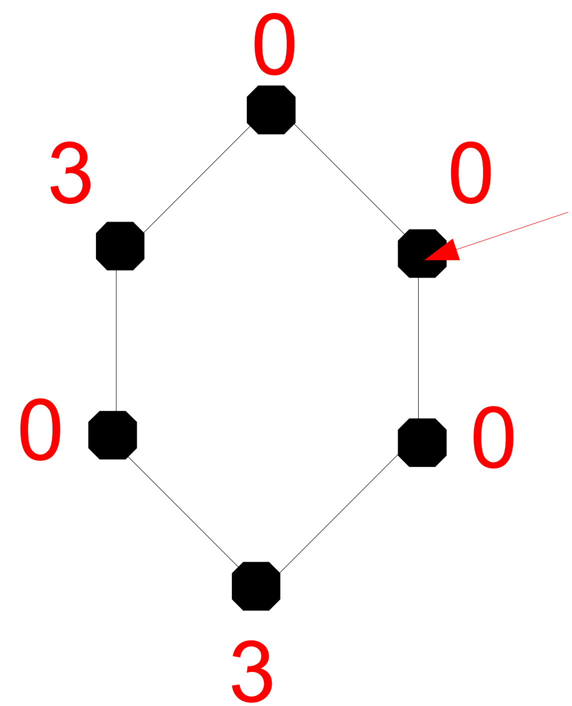
Solvable Configurations

- A configuration is solvable if every vertex could be reached by a pebble after pebbling moves

Solvable



Unsolvable



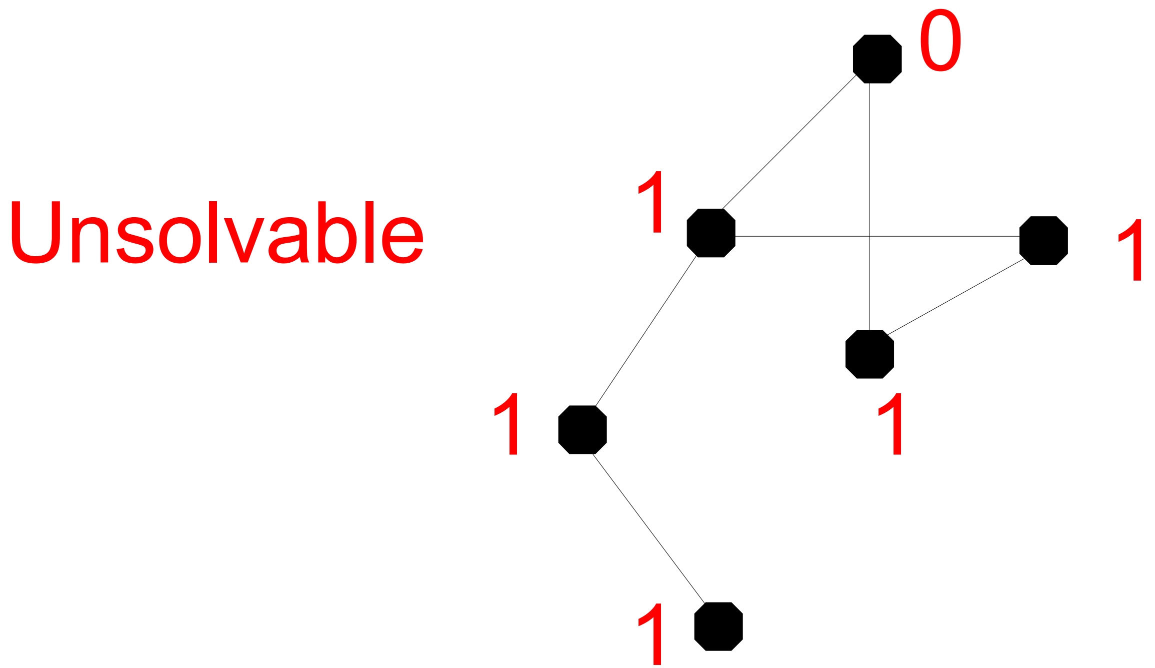
Pebbling Number

- The pebbling number of a graph, $\pi(G)$, is the number of pebbles needed to guarantee that all configurations of $\pi(G)$ pebbles are solvable
- Question: Determine or set bounds on $\pi(G)$ that are determined by characteristics of G.

Basic Results

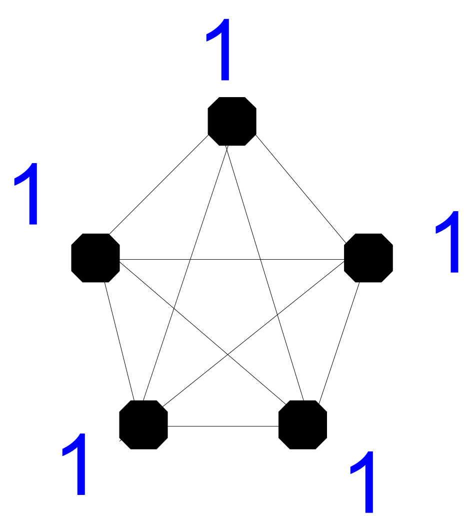
$\pi(G) \geq |V(G)|$

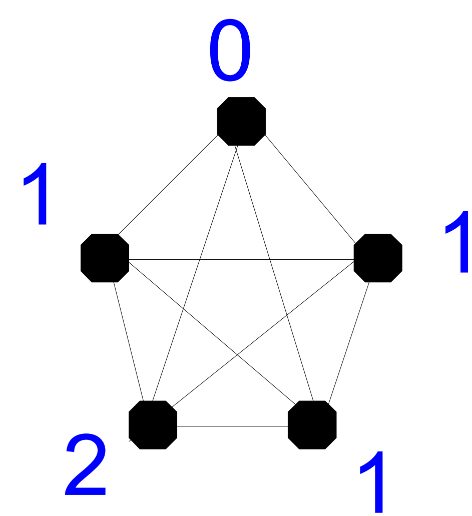
- Otherwise, the configuration with one pebble at every vertex except for one is unsolvable



Basic Results

$\pi(K_n) = |V(G)|$, where K_n is the complete graph on n vertices

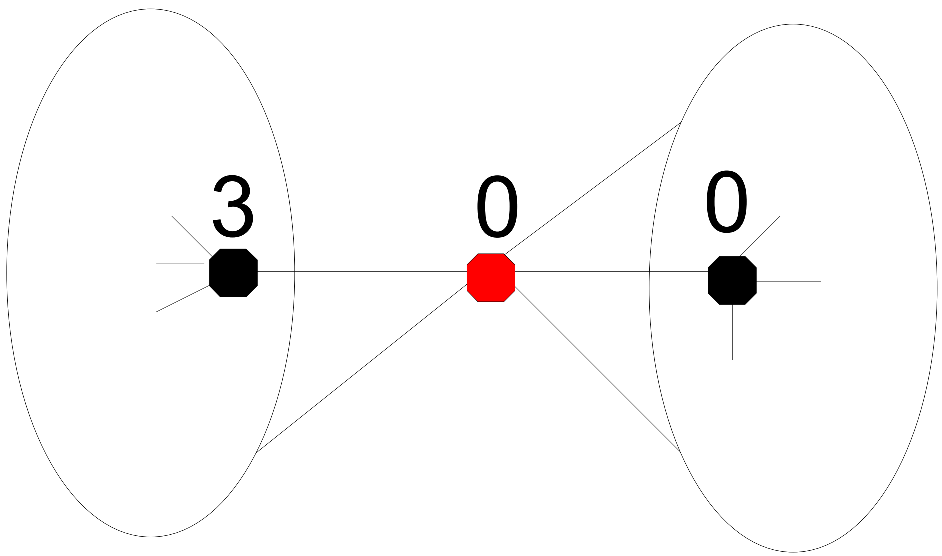




- Configurations with a vertex with no starting pebbles means one vertex has at least 2 pebbles by pigeon hole, edge exists since complete graph

Basic Results

π(G) > |*V(G)*| if *G* contains a cut vertex



- Place 1 pebble at every vertex besides for the vertices around the cut as shown. This configuration is unsolvable with |*V(G)*| pebbles

Problems

- Determine $\pi(G)$ for certain families of graphs
- Classify all graphs that have $\pi(G) = |V(G)|$ and all graphs that have a certain number of subgraphs that satisfy this property
- Examine in more generality: weighted pebbling moves, cover pebbling