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Finding Hamiltonian Paths in Cocomparability Graphs Using the Bump Number Algorithm*

PETER DAMASCHKE, JITENDER S. DEOGUN, DIETER KRATSCH, and GEORGE STEINER

Friedrich-Schiller-Universität Jena, Mathematische Fakultät, Universitätshochhaus. 17. OG, O6900 Jena, Germany; Department of Computer Science & Engineering, University of Nebraska – Lincoln, Lincoln, NE 68588-0115, U.S.A.; Management Science and Information Systems Area, Faculty of Business, McMaster University, Hamilton, Ontario, Canada L8S 4M4

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Abstract. *Hamiltonian Path/Cycle* are well known NP-complete problems on general graphs, but their complexity status for permutation graphs has been an open question in algorithmic graph theory for many years. In this paper, we prove that the *Hamiltonian Path* problem is solvable in polynomial time even for the larger class of cocomparability graphs. Our result is based on a nice relationship between Hamiltonian paths and the bump number of partial orders. As another consequence we get a new interpretation of the bump number in terms of path partitions, leading to polynomial time solutions of the *Hamiltonian Path/Cycle Completion* problems in cocomparability graphs.

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1. Introduction and Notation

Let $G = (V, E)$ be an undirected graph, where $V = \{v_1, v_2, \ldots, v_n\}$ is a finite set of vertices and let Π_V be the set of all permutations of the elements $v_1, v_2, \ldots, v_n$. The Hamiltonian Path completion number of a permutation $\pi = (v_1, v_2, \ldots, v_n) \in \Pi_V$ with respect to a graph $G = (V, E)$ is

$$h_p(\pi, G) = |\{v_i : (v_{i-1}, v_i) \notin E\}|$$

and the Hamiltonian path (HP) completion number of graph G is

$$h_p(G) = \min\{h_p(\pi, G) : \pi \in \Pi_V\}.$$

The Hamiltonian Cycle completion number of permutation $\pi = (v_1, v_2, \ldots, v_n) \in \Pi_V$ with respect to a graph $G = (V, E)$ is

$$\begin{aligned} h_c(\pi, G) &= h_p(\pi, G) && \text{if } (v_n, v_1) \in E, \text{ and} \\ h_c(\pi, G) &= h_p(\pi, G) + 1 && \text{otherwise.} \end{aligned}$$

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The Hamiltonian Cycle (HC) completion number of graph G is defined as

$$h_c(G) = \min\{h_c(\pi, G): \pi \in \Pi_V\}.$$

When $h_p(\pi, G) = h_p(G)$, π is called an optimal path completion and when $h_c(\pi, G) = h_c(G)$, π is called an optimal cycle completion. When $h_p(\pi, G) = 0$, π is called a Hamiltonian path (HP) and when $h_c(\pi, G) = 0$, π is called Hamiltonian Cycle (HC) and the graph G is called a Hamiltonian Graph.

A partial order will be denoted by $P = (V, <_P)$, where V is the finite ground set of elements or vertices and $<_P$ is an irreflexive, antisymmetric, and transitive binary relation on V. Two elements $a, b \in V$ are comparable in P (denoted by $a \sim_P b$) if $a <_P b$ or $b <_P a$. Otherwise, they are said to be incomparable (denoted by $a \parallel b$).

A linear order is a partial order without incomparable elements. A linear extension of a partial order $P = (V, <_P)$ is a linear order $L = (V, <_L)$ on the same ground set that extends P, i.e.,

$$a <_P b \Rightarrow a <_L b \quad \text{for all } a, b \in V.$$

We will write linear orders as $L = v_1, v_2, \ldots, v_n$ where the sequence from left to right defines the order relation $<_L$, i.e., $v_1 <_L v_2 <_L \cdots <_L v_n$.

Let $L = v_1, v_2, \ldots, v_n$ be a linear extension of $P = (V, <_P)$. Two consecutive elements v_i, v_{i+1} of L are separated by a bump if $v_i <_P v_{i+1}$. The total number of bumps in L is denoted by $b(L, P)$.

The bump number $b(P)$ of a partial order P is the minimum number of bumps in some linear extension, i.e.,

$$b(P) = \min\{b(L, P) \mid L \text{ is a linear extension of } P\}.$$

A linear extension L of P with $b(L, P) = b(P)$ is called an optimal linear extension.

The graph $G = (V, E)$, edges of which are exactly the comparable pairs of a partial order P on V, is called the comparability graph of P, and is denoted by $G(P)$. The complementary graph, whose edges are incomparable pairs in P, is called the cocomparability graph of P and will be denoted by $G^c(P)$. Alternatively, a graph G is a cocomparability graph, if its complementary graph G^c has a transitive orientation, corresponding to the comparability relations of a partial order, denoted by P_G. We note that a partial order uniquely determines its comparability graph $G(P)$ and its cocomparability graph $G^c(P)$, but the reverse is not true, i.e., a cocomparability graph G has as many partial orders P_G, as is the number of the transitive orientations of G^c.

A path $\pi = (v_1, v_2, \ldots, v_n) \in G^c(P)$ is monotone if $v_i <_P v_j$ implies $i < j$. So the notions of monotone Hamiltonian path and bump free linear extension are equivalent.

Deciding whether $h_p(G) = 0$ ($h_c(G) = 0$) are the well known *Hamiltonian Path* (*Cycle*) problems, that are NP-complete even when restricted to several special classes of graphs ([1, 2, 5, 6, 11]). In particular, these problems remain NP-complete for bipartite graphs [15] and hence for comparability graphs.

In Section 2, we prove that *Hamiltonian Path* is polynomial-time solvable for cocomparability graphs, and consequently for permutation graphs, i.e., the comparability (co-comparability) graphs of 2-dimensional partial orders. Cocomparability graphs represent a naturally arising class of graphs where we can look for polynomial solutions of problems which are NP-complete for general graphs ([14]). Polynomial algorithms were known before only for a few special cases including cographs [3], interval graphs, i.e., cocomparability graphs of interval orders [4, 13], convex bipartite graphs [6] and bipartite permutation graphs [2]. An involved polynomial algorithm is given in [4] for the class of circular arc graphs. A complex, polynomial-time algorithm is given in [7] for *Hamiltonian Cycle* in cocomparability graphs.

The more general problems of determining $h_p(G)$ and $h_c(G)$ are closely related to the *Partition into Paths* (PIP) problem:

Given a graph $G = (V, E)$, and an integer k: is there a partition of the vertex set of G into at most k paths? PIP is polynomial-time solvable for interval graphs and circular arc graphs [4].

In this paper we present a polynomial time solution for finding a HP or $h_p(G)$ and $h_c(G)$ for a cocomparability graph G. Most of our results are based on exploiting the close relationship of these problems to the problem of finding the bump number of a partial order.

Habib, Möhring, and Steiner [10] and independently Schäffer and Simons [16] have found polynomial time algorithms for computing the bump number of general partial orders. Here we use the results in [10].

The generalized weak order P^* belongs to the class $P_{r,s}$ if it is a parallel composition of s isolated elements and the series composition of r antichains $A_1, A_2, \ldots, A_r$. If $P^* \in P_{r,s}$ then $b(P^*) = \max\{0, r-1-s\}$.

The HMS algorithm [10] is of primal-dual type. Primal solutions are linear extensions of the given order P and the dual solutions correspond to generalized weak orders contained in P. Containment, here is defined with respect to inclusion of the order relations, i.e., P^* is contained in P or equivalently, P extends P^*, if $a <_{P^*} b \Rightarrow a <_P b$ for all $a, b \in V$. Let L be any linear extension of the given P and let P^* be any generalized weak order contained in P, then

$$b(P^*) \leqslant b(P) \leqslant b(L, P).$$

The HMS algorithm constructs L and P^* such that $b(P^*) = b(L, P)$ thus implying equality throughout the above inequality.

The generalized weak order produced by the algorithm always satisfies

$$s < r.$$

Thus for the generalized weak order P^* produced by the algorithm, we have $b(P^*) = r - 1 - s$. Now $b(P)$ can be zero if and only if $r - 1 - s = 0$. So if $b(P) = 0$ then $r = s + 1$.

In Theorem 1, we summarize the main results from [10] in a format suitable for our use:

THEOREM 1 [10]. *For any partial order P*

1. $\max\{b(P^*) \mid P^* \subseteq P, P^*$ *a generalized weak order*$\} = \min\{b(L, P) \mid L$ *is a linear extension of* $P\}$.
2. *There is an algorithm running in time* $O(n^2)$ *which finds a linear extension L of a given poset P with* $b(L, P) = b(P)$, *and hence, computes* $b(P)$.

2. Hamiltonian Paths and Cycles in Cocomparability Graphs and the Bump Number

We define the concept of graph containment as follows: A graph $G = (V, E)$ is said to *contain* a graph $G' = (V', E')$ if $V' = V$ and $E' \subseteq E$. The following monotonicity properties can be easily proven: If G contains G' then $h_p(G) \geqslant h_p(G')$ and $h_c(G) \geqslant h_c(G')$.

THEOREM 2. *If* $G = (V, E)$ *is a cocomparability graph then*

1. $h_p(G) = b(P_G)$.
2. $b(P_G) \leqslant h_c(G) \leqslant b(P_G) + 1$.

Proof. It is well known [8, 9] that the bump number is a comparability invariant, i.e., if we have two different partial orders P_G with the same comparability graph then their bump number will be the same too.

1. Consider an optimal linear extension L of P_G. If for every bump $v_i <_P v_{i+1}$ in L we add the edge (v_i, v_{i+1}) to G then the sequence $v_1, v_2, \ldots, v_n$ represents a path completion of G. Thus, we have $h_p(G) \leqslant b(L, P_G) = b(P_G)$. If $b(P_G) = 0$ then this automatically implies $h_p(G) = 0$, i.e., G has a Hamiltonian path. For $b(P_G) > 0$ it was shown in [10] that there exists a generalized weak order $P_{r,s}$ contained in P_G for which $b(P_G) = b(P_{r,s}) = r - 1 - s$ [cf. Theorem 1 (1)]. The cocomparability graph of $P_{r,s}$, $G^c(P_{r,s})$ is the disjoint union of r cliques and an additional clique on s elements, each of which is adjacent to every point in the r cliques. It is easy to see that $h_p(G^c(P_{r,s})) = r - 1 - s > 0$. On the other hand $P_{r,s}$ is contained in P_G, so $G^c(P_{r,s})$ contains G. By the above monotonicity property $h_p(G) \geqslant h_p(G(P_{r,s})) = r - 1 - s = b(P_G)$, completing the proof of 1.

2. It is clear that any optimal path completion of G can be transformed into a cycle completion of G by connecting the start and end point of an optimal path by an edge. Thus we always have $h_p(G) \leqslant h_c(G) \leqslant h_p(G) + 1$. In view of part 1 this proves 2. □

COROLLARY 1. *Hamiltonian Path and Hamiltonian Path Completion are solvable in* $O(n^2)$ *time for a cocomparability graph G and an HP (or a minimum HP completion if no HP exists) can be found in* $O(n^2)$ *time.*

Proof. By Theorem 1 it suffices to find an optimal linear extension L with $b(L, P_G) = b(P_G)$: If $b(P_G) = 0$ then L is an HP, if $b(P_G) > 0$ then G has no HP, and L is a minimum HP completion. □

3. A Reformulation and Direct Proof

In this section we present a direct proof of Theorem 2 which has the advantage that it does not refer to Theorem 1. It "shows what happens" in a more transparent way. Theorem 3 below is a reformulation of Theorem 2 that leads to a direct proof of Theorem 2.

THEOREM 3. *Let $G = (V, E)$ be a cocomparability graph.*
1. *G has a HP iff G has a monotone HP.*
2. *$b(P_G) = \min\{k$: G can be partitioned into k paths$\} - 1$.*

Note that this is indeed only a reformulation of Theorem 2. First we prove a lemma.

LEMMA 1. *Let $(v_1, \ldots, v_n)$ be a HP in G such that v_1 is a minimal element of $P = P_G$. Then there exists a monotone HP starting with the same vertex v_1.*

Proof. We describe a procedure that rebuilds the given HP into a monotone HP. The procedure works in n steps where the vertices are renumbered. After the i-th step, the following propositions will hold:
1. $(v_1, \ldots, v_i)$ is a HP in the subgraph of G induced by $\{v_1, v_2, \ldots, v_i\}$.
2. $(v_1, \ldots, v_i)$ is monotone.
3. There is no pair (l, m) with $l \leqslant i$, $m \geqslant i+1$, and $v_m <_P v_l$.

In the first step we do nothing. Since v_1 is minimal, (1)–(3) are trivially true. Suppose that (1)–(3) hold after the i-th step. We describe the $(i+1)$-th step, and show that (1)–(3) remain valid for $i := i+1$.

We shall not modify the subpath $(v_1, \ldots, v_i)$. Therefore (3) implies that (2) remains true also for $i := i+1$, independently from the choice of the new v_{i+1}. So we must show only (1) and (3).

We define $C = \{v_j : j > i+1, v_{i+1} >_P v_j\}$. If $C = \emptyset$ then (1) and (3) are already true, and we do nothing. Otherwise let v_k be the rightmost vertex of C, i.e., k is as large as possible. $v_i >_P v_k$ is impossible due to (3), $v_i <_P v_k$ would imply $v_i <_P v_k <_P v_{i+1}$ which contradicts $(v_i, v_{i+1}) \in E$. So we have $(v_i, v_k) \in E$. Assume $k < n$, then $v_{i+1} >_P v_{k+1}$ would contradict the choice of v_k, and $v_{i+1} <_P v_{k+1}$ would imply $v_k <_P v_{i+1} <_P v_{k+1}$, which contradicts $(v_k, v_{k+1}) \in E$. So the only possibility is $(v_{i+1}, v_{k+1}) \in E$.

Now we set $v_{i+1} := v_k, v_{i+2} := v_{k-1}, \ldots, v_k := v_{i+1}$, i.e., we reverse the subpath $(v_{i+1}, \ldots, v_k)$ Since $(v_i, v_k) \in E$, and (in case $k < n$) $(v_{i+1}, v_{k+1}) \in E$, (1) is ensured. Further we know that the new v_{i+1} is properly smaller (with respect to

$<_P$) than the old v_{i+1}. So we can repeat this transformation only finitely many times. After termination, the actual v_{i+1} is minimal in $(v_{i+1}, \ldots, v_n)$, thus (3) is satisfied for $i+1$ instead of i. This completes the $(i+1)$-th step.

After the n-th step, $(v_1, \ldots, v_n)$ is a monotone HP. Finally note that v_1 is still the original v_1. This proves the lemma. □

Proof of Theorem 3.

1. Let $(v_1, \ldots, v_n)$ be an arbitrary HP in G. Let k be the smallest index such that v_k is minimal or maximal. W.l.o.g. suppose that v_k is minimal in P_G, otherwise we consider the dual poset P_G^d. We apply our lemma to the subpath $(v_k, \ldots, v_n)$, and we obtain a new HP $(v_1, \ldots, v_n)$ where the subpath $(v_k, \ldots, v_n)$ is monotone. Since $(v_1, \ldots, v_{k-1})$ contains no maximal element, v_n is a maximal element in P_G (and not only in the subposet induced by $\{v_k, \ldots, v_n\}$!). Now we consider the reversed HP $(v_n, \ldots, v_1)$ in P_G^d, v_n is a minimal element, so we can apply our lemma again, thereby obtaining a monotone HP.

2. Trivially, G can be partitioned into $b(P_G)+1$ paths. It remains to show that there is no partition into fewer paths. Consider a partition of G into k paths. Add to P_G exactly $k-1$ new elements, each of which is incomparable to all other elements. We denote this augmented poset by Q. One can easily see $b(Q) = \max\{0, b(P_G) - (k-1)\}$. On the other hand, the cocomparability graph of Q has a HP: Since the $k-1$ new vertices are adjacent to all other ones, the k paths of our partition can be linked to a HP. By Part 1, we have $b(Q) = 0$, thus $b(P_G) \leqslant k-1$, hence $k \geqslant b(P_G)+1$, as asserted. □

Theorem 3 yields a new interpretation of the bump number. As a byproduct, we get a new proof for the already known fact that the bump number is a comparability invariant, i.e., $b(P_G)$ depends only on G.

4. Duality

Let us call a graph $G = (V, E)$ a *clique composition* (CC) graph if there is a partition $V = V_0 \cup V_1 \cup \cdots \cup V_r$ such that

1. each V_i is a clique;
2. there exists no $(u, v) \in E$ with $u \in V_i, v \in V_j$ for $1 \leqslant i < j \leqslant n$;
3. for every $u \in V_0$ and $v \in V_j (1 \leqslant j \leqslant r)(u, v) \in E$.

In other words, CC graphs can be built by taking $r+1$ cliques and connecting every element of one distinguished clique (V_0) to every element in the other cliques. If $s = |V_0|$ for such a CC graph then we will say that G is from the class $C_{r,s}$ of *clique composition* graphs. (Note that we allow $s = 0$, but we always assume $|V_i| > 0$ for $i \geqslant 1$). It can be easily seen that $C_{r,s}$ is exactly the class of cocomparability graphs $\{G^c(P)\}$ for the generalized weak orders $P \in \mathscr{P}_{r,s}$ defined in [10]. Interestingly, as the class $\mathscr{P}_{r,s}$ played a strong duality role to solutions of the bump number problem, the class $\mathscr{C}_{r,s}$ plays a similar role in the Hamiltonian completion problems, we are dealing with.

The monotonicity properties in Section 2 immediately yield the following "weak duality" result:

LEMMA 2. *Let $G = (V, E)$ be a cocomparability graph.*

1. *For any path completion π of G and any CC graph G^* containing G*

$$h_p(G^*) \leqslant h_p(G) \leqslant h_p(\pi, G);$$

2. *For any cycle completion π of G and any CC graph G^* containing G*

$$h_c(G^*) \leqslant h_c(G) \leqslant h_c(\pi, G).$$

Since the clique composition graphs in $\mathscr{C}_{r,s}$ are exactly the cocomparability graphs of the generalized weak orders in $\mathscr{P}_{r,s}$, in view of Theorem 2, Theorem 1 (part 1) above is equivalent to the following "strong duality" result for Hamiltonian path completion in cocomparability graphs.

THEOREM 4. *For any cocomparability graph $G = (V, E)$*

$$\max\{h_p(G^*) \mid E^* \supseteq E, G^* = (V, E^*) \textit{ is a CC graph}\}$$
$$= \min\{h_p(\pi, G) \mid \pi \textit{ is a path completion of } G\}.$$

Let us call a graph $G = (V, E)$ *h_p-maximal* if for any edge $e \in V \times V - E$, the Hamiltonian path completion $h_p(G + e) < h_p(G)$, where $G + e$ is the graph on V with edge set $E \cup e$. A graph G is defined to be h_c-maximal in similar way. These definitions are meaningful because of the monotonicity properties.

It can be easily seen that for any $G^* \in \mathscr{C}_{r,s}$

$$h_p(G^*) = \max\{0, r-1-s\}, \quad \text{and}$$

$$h_c(G^*) = \begin{cases} \max\{0, r-s\} & \text{if } r > 1, \\ 0 & \text{if } r = 1, s = 0. \end{cases}$$

It is clear that every $G^* \in \mathscr{C}_{r,s}$ with $r - 1 > s$ is both h_p-maximal and h_c-maximal. If $r = 1$, $s = 0$ and $G^* \in \mathscr{C}_{r,s}$ then G^* is a clique itself and can be considered both h_p-maximal and h_c-maximal, as it vacuously satisfies the definitions. If $r - 1 = s > 0$, then for any $G^* \in \mathscr{C}_{r,s}$, it follows that $h_p(G^*) = 0$ and $h_c(G^*) = 1$ and G^* is h_c-maximal. Theorem 4 then yields:

THEOREM 5. *A cocomparability graph $G = (V, E)$ is h_p-maximal iff $G \in \mathscr{C}_{r,s}$ with $r = 1$ and $s = 0$ (then G is a clique) or with $r - 1 > s$ (then $h_p(G) = r - 1 - s$).*

5. Hamiltonian Cycle Completion

In this section first we deal with the determination of the Hamiltonian cycle completion number, when the bump number of a confirming partial order is known to be non-zero.

THEOREM 6. *Let* $G = (V, E)$ *be a cocomparability graph and* $b(P_G) > 0$ *then* $h_c(G) = b(P_G) + 1$.

Proof. From Theorem 2 we have $b(P_G) \leqslant h_c(G)$. So $h_c(G) > 0$. Let π be an optimal cycle completion. Let $\pi = (v_{\iota_1}, v_{\iota_2}, v_{\iota_3}, \ldots, v_{\iota_n})$. Since $h_c(G) > 0$ there is at least one pair of vertices in the optimal permutation π which doesn't describe an edge of the graph. W.l.o.g. we can assume that $(v_{\iota_n}, v_{\iota_1}) \notin E$, and it follows that $h_p(\pi, G) = h_c(\pi, G) - 1$.

As the Hamiltonian path completion number is minimum over all permutations, we have

$$h_p(G) \leqslant h_c(\pi, G) - 1 = h_c(G) - 1.$$

From Theorem 2, we have $h_c(G) \leqslant b(P_G) + 1 = h_p(G) + 1$. Thus it follows that

$$h_c(G) = b(P_G) + 1. \qquad \square$$

Now we know that $h_c(G) = b(P_G) + 1$ whenever $b(P_G) > 0$. It remains to consider whether $h_c(G) = b(P_G)$ or $h_c(G) = b(P_G) + 1$ when $b(P_G) = 0$. i.e., to determine whether $h_c(G) = 0$ or 1 when $b(P_G) = 0$. This of course is equivalent to solving the *Hamiltonian Cycle* problem in cocomparability graphs, whose complexity status has been open [12]. This question was settled in [7] by the following theorem, which has a very long proof there.

THEOREM 7. [7]. *Let us consider a cocomparability graph G with* $b(P_G) = 0$. *If* P_G *has a (critical) element* x *whose deletion increases the bump number (to* $b(P_G - x) = 1$) *then* $h_c(G) = 1$, *otherwise* $h_c(G) = 0$.

In view of Theorems 2, 5, *and* 6, *this also yields the following.*

COROLLARY 2. *A cocomparability graph* $G(V, E)$ *is* h_c*-maximal iff* $G \in \mathscr{C}_{r,s}$ *with* $r = 1$ *and* $s = 0$ *(then G is a clique and* $h_c(G) = 0$), *or with* $r - s = 1$ *and* $s > 0$ *(then* $h_c(G) = 1$ *and* $h_p(G) = 0$) *or with* $r - s > 1$ *(then* $h_c(G) = r - s$ *and* $h_p(G) = r - s - 1$).

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