Proof a Maximal Antichain is a Quantum Cover

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Introduction

<u>Quantum Measure</u>: a function μ:P(X) $\rightarrow \mathbb{R}^+$ such that:

1. Grade 2 Additive:
   – If A, B, and C are mutually disjoint, then:

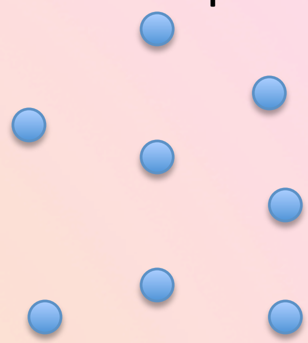
$$\mu(A \cup B \cup C) = \mu(A \cup B) + \mu(A \cup C) + \mu(B \cup C) + \mu(A) - \mu(B) - \mu(C)$$

2. Regular:
   ① If $\mu(A) = 0$, then for all set $B \sqcup A$, $\mu(A \cup B) = \mu(B)$
   ② If $A \cap B = \varnothing$ and $\mu(A \cup B) = 0$, then $\mu(A) = \mu(B)$

Introduction

<u>Cover</u>: A collection of sets $A_i \in P(X)$ is a cover for X if $A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_i = X$.

Example:



<u>Quantum Cover</u>: if $\{A_i\}$ is a cover for X, then if $\mu(A_i) = 0$ for all ‘i’, then $\mu(X) = 0$

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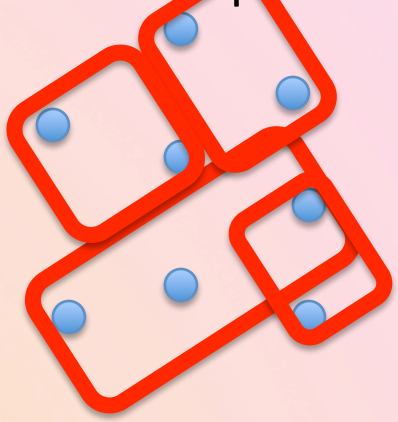

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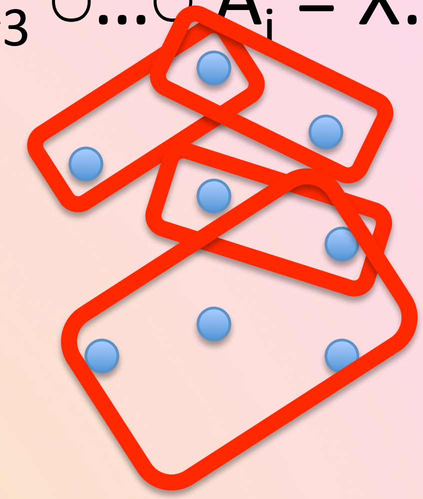

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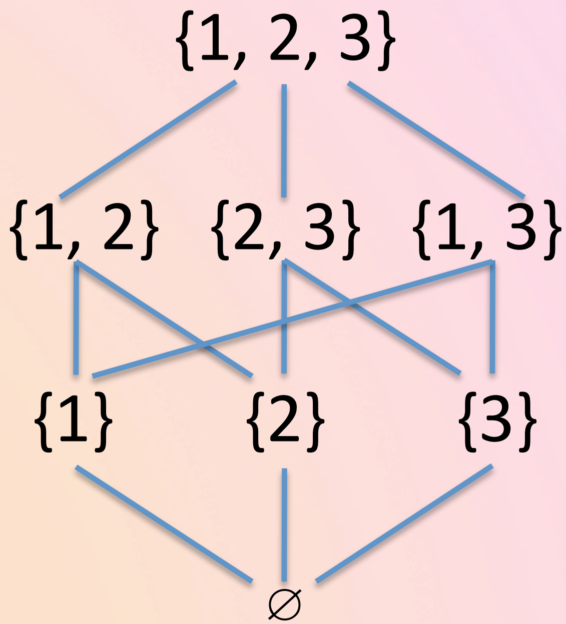
Antichain: a collection of sets such that none of the sets contain each other (the elements can overlap)



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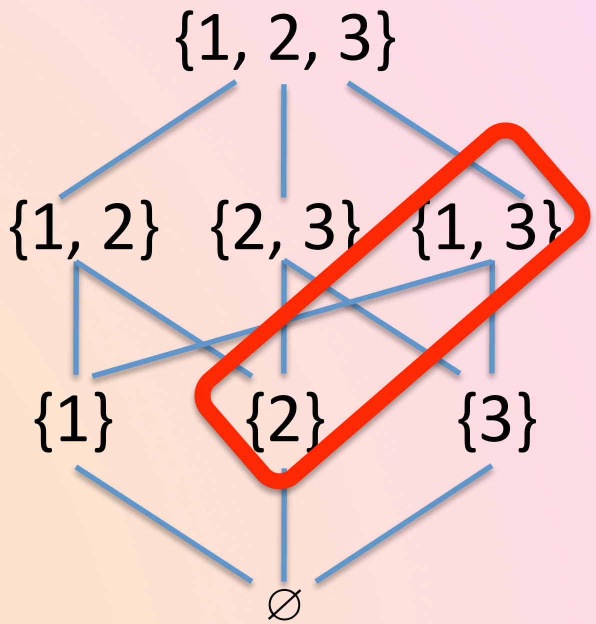

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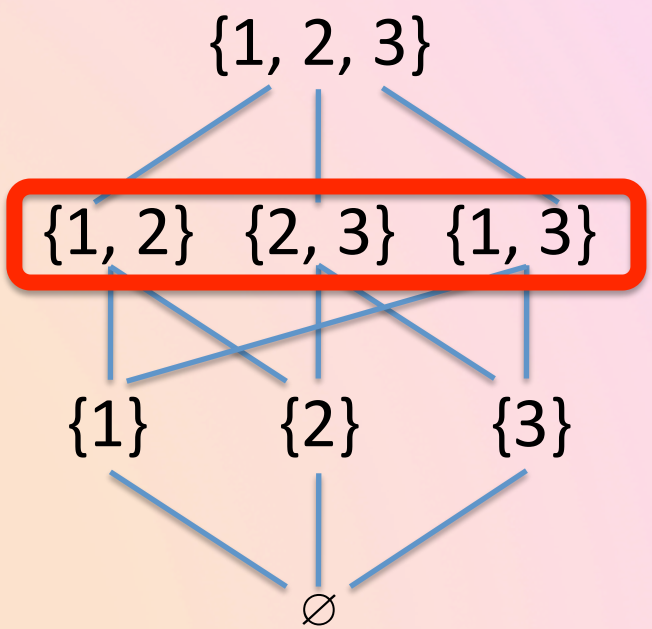

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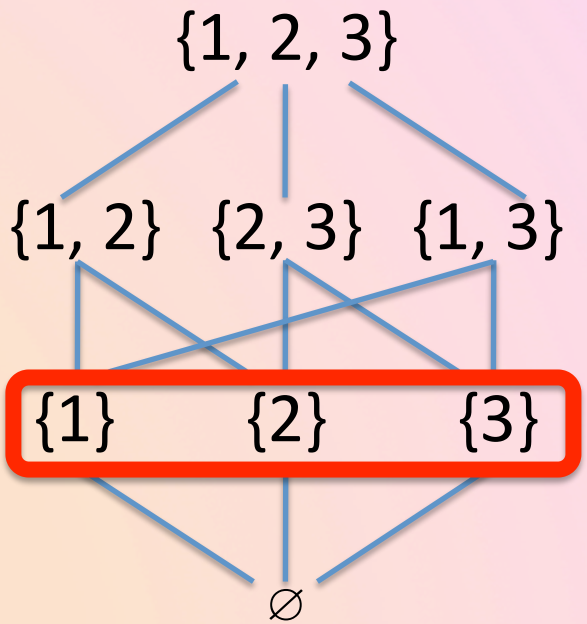

Conjecture

- Every maximal antichain is a quantum cover.
  - If the maximal antichain has measure of zero (0), then the measure of the set is zero (0).

Approach

- Understand what has been done
- Build on from what has been done
- Proof for small cases

Citation

- S. Gudder, Finite Quantum Measure Spaces, *Amer Math Monthly 177 (2010) 512—527*

- S. Surya and P. Wallden, Quantum covers in quantum measure theory (2008)