Are all Maximal Antichains Quantum Covers?

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Introduction

<u>Quantum Measure</u>: a function $\mu: P(X) \to \mathbb{R}^+$ that is:

1. Grade 2 Additive:
   – If A, B, and C are mutually disjoint, then:

     $\mu(A \cup B \cup C) = \mu(A \cup B) + \mu(A \cup C) + \mu(B \cup C) + \mu(A) - \mu(B) - \mu(C)$

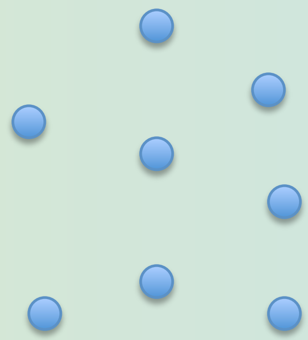
2. Regular:
   ① If $\mu(A) = 0$, then for all sets B such that $A \cap B = \emptyset$, $\mu(A \cup B) = \mu(B)$

   ② If $A \cap B = \emptyset$ and $\mu(A \cup B) = 0$, then $\mu(A) = \mu(B)$

Introduction

Cover: A collection of sets $\{A_1, A_2, \ldots, A_i\}$ is a cover for X if $A_1 \cup A_2 \cup \ldots \cup A_i = X$.

Example:



Quantum Cover: A cover $\{A_1, A_2, \ldots, A_i\}$ is a quantum cover if for any quantum measure μ if $\mu(A_i) = 0$ for all ‘i’, imples $\mu(X) = 0$

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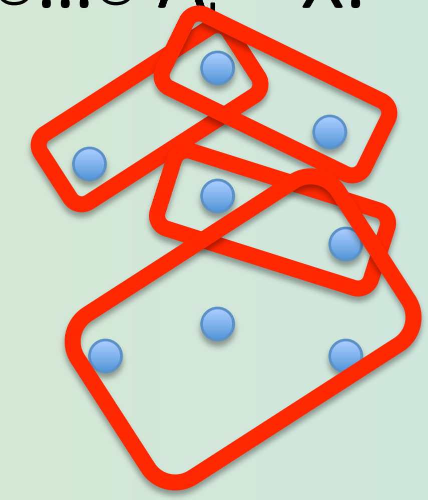

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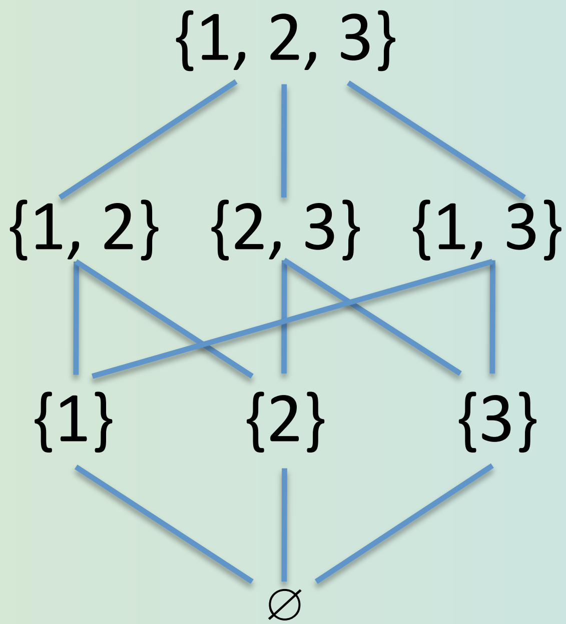
Antichain: a collection of sets such that none of the sets contain each other (their elements can overlap)



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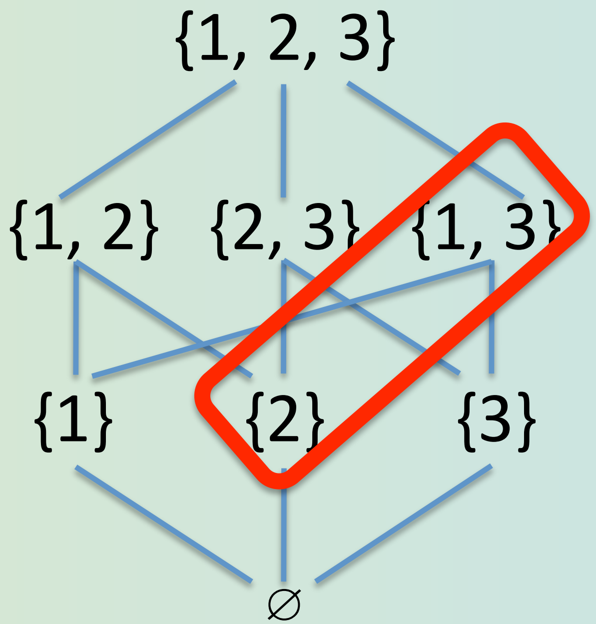

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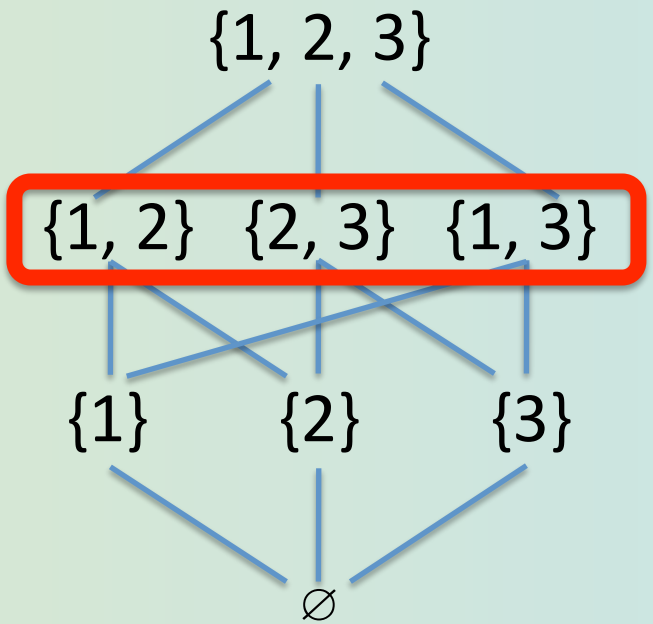


Conjecture

- Every maximal antichain is a quantum cover.
  - For any quantum measure μ, if all sets in maximal antichain have measure zero (0), then the measure of the whole set is zero (0).
- In order to build intuition, studied triple slit experiment.

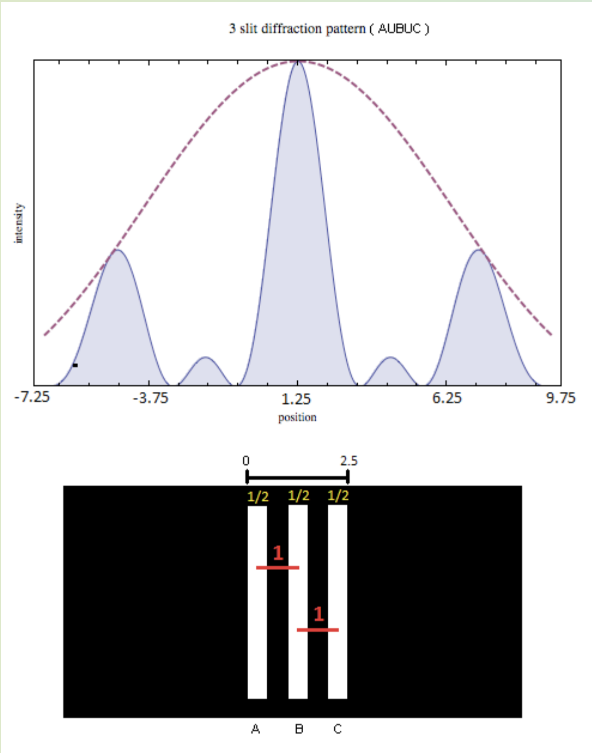
Application: Triple Slit Experiment

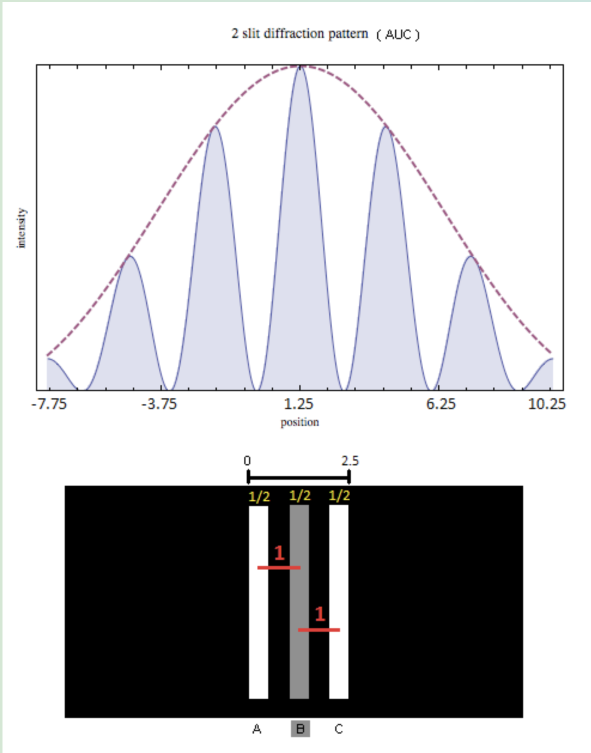
•Used Fraunhofer diffraction equation to model the intensity of single, double, and triple slit experiment

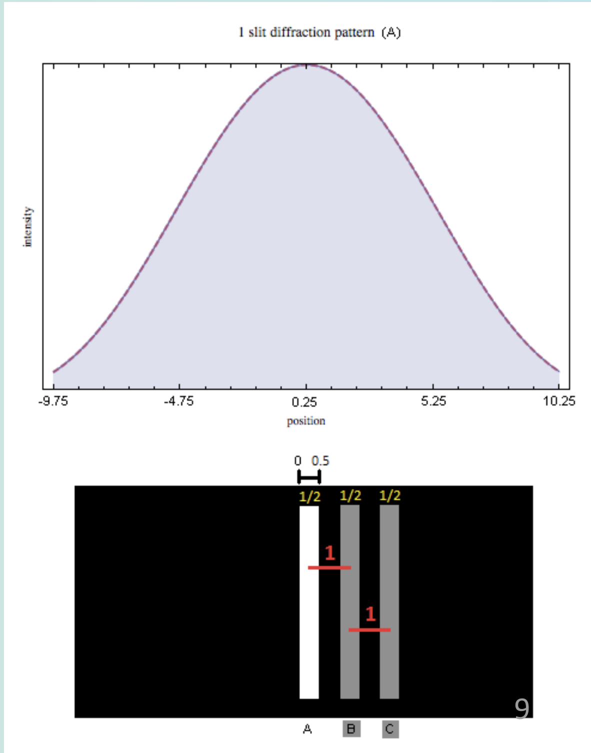
$$\frac{I}{I_0} = \left(\frac{1}{n} U_{n-1}\left(\cos\left(\frac{d\,q}{2}\right)\right) \operatorname{sinc}\left(\frac{a\,q}{2}\right)\right)^2$$

- d = distance between slits
- a = slit's width
- Parameter q is normalized wavenumber

○http://demonstrations.wolfram.com/MultipleSlitDiffractionPattern/

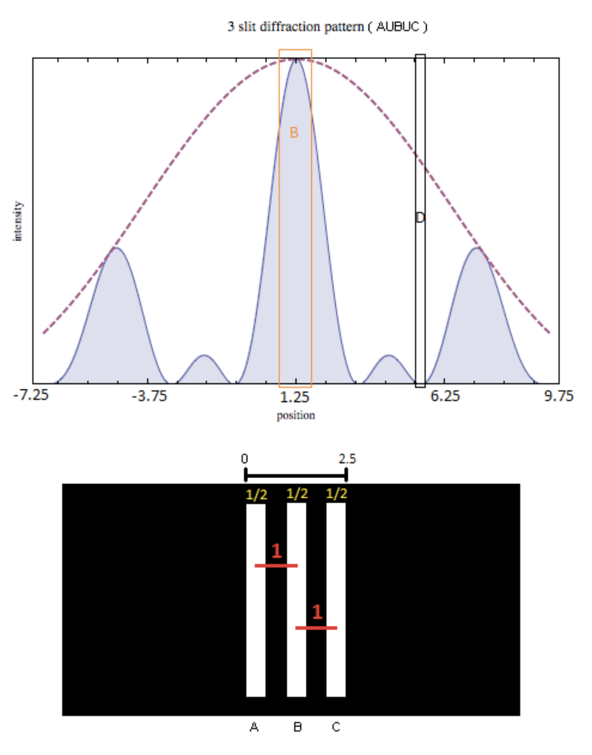


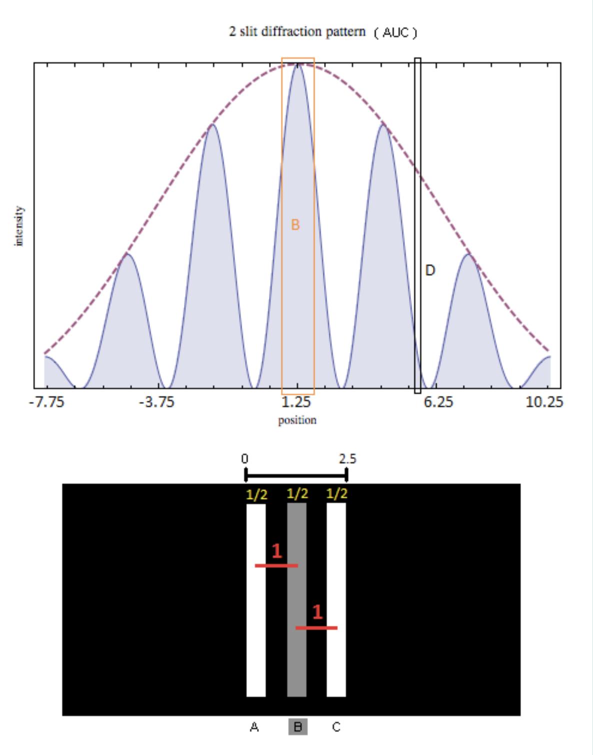


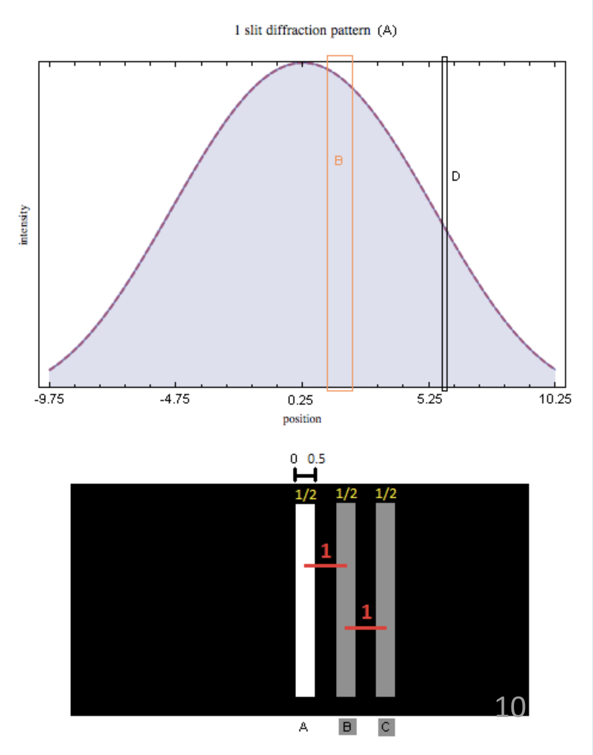


Application: Triple Slit Experiment

•In this experiment, a quantum measure specifies the probability that an electron lands in a given region







Example Data

B region	1.25	2.25
D region	5.9	5.95

Error:		
	B region	-0.04228
	D region	-0.00412

AUBUC	Whole area	3.75852		
Center:	B region area	0.803756	B probability	0.213849068
1.25	D region area	0.00295094	D probability	0.000785134

AUC	Whole area	5.48544		
Center:	B region area	0.723667	B probability	0.13192506
1.25	D region area	4.98577E-05	D probability	9.0891E-06

AUB	Whole area	5.64631		
Center:	B region area	0.743514	B probability	0.131681399
0.75	D region area	0.0199846	D probability	0.003539409

BUC	Whole area	5.64631		
Center:	B region area	0.977756	B probability	0.173167254
1.75	D region area	0.00836763	D probability	0.001481964

A	Whole area	11.0806		
Center:	B region area	0.952451	B probability	0.085956627
0.25	D region area	0.0242706	D probability	0.002190369

B	Whole area	11.0806		
Center:	B region area	0.99309	B probability	0.089624208
1.25	D region area	0.0309991	D probability	0.002797601

C	Whole area	11.0806		
Center:	B region area	0.99309	B probability	0.089624208
2.25	D region area	0.0374229	D probability	0.003377335

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Analysis

- Are all maximal antichains μ-specific quantum covers?
  - **<u>μ-specific quantum cover</u>**: A cover $\{A_1, A_2, \ldots, A_i\}$ is a μ-specific quantum cover if for a specific quantum measure μ, $\mu(A_i) = 0$ for all 'i' implies $\mu(X) = 0$

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Future Approach

- Study error
- Try to get experimental data
- Look at four and five slit experiments

Citation

- C. Couteau, T. Jennewein, R. Laflamme, U. Sinha, and G. Weihs, Ruling Out Multi-Order Interference in Quantum Mechanics, Science329 (2010) 418—421

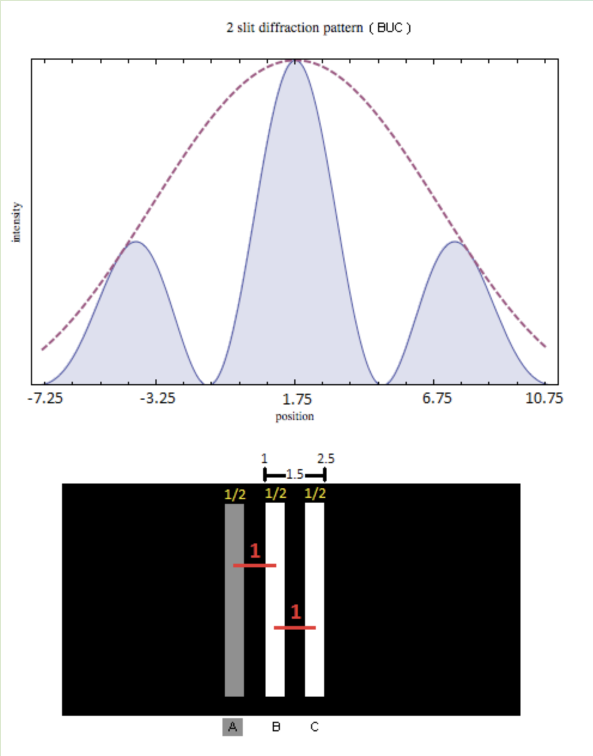
- “Multiple Slit Diffraction Pattern” from the Wolfram Demonstrations Porject http://demonstrations.wolfram.com/MutipleSlitDiffractionPattern/

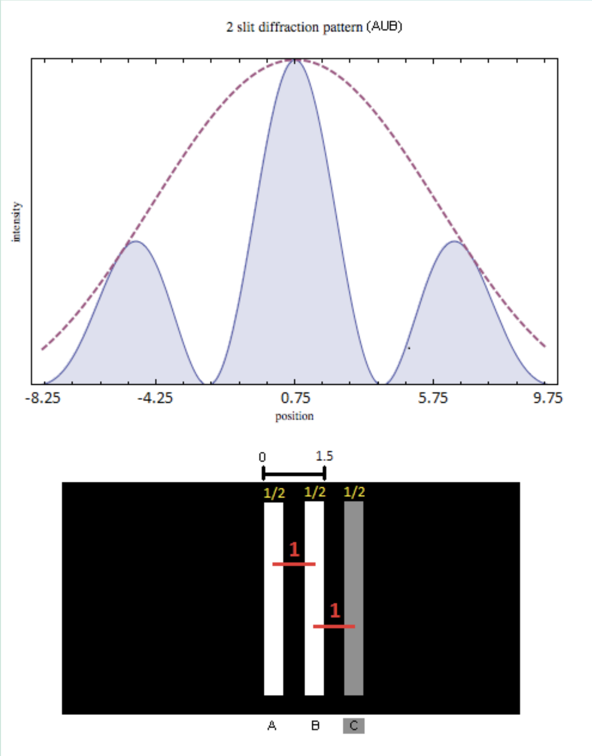
- S. Gudder, Finite Quantum Measure Spaces, Amer Math Monthly 177 (2010) 512—527

- S. Surya and P. Wallden, Quantum covers in quantum measure theory (2008)

Thank you!

Extra Diagrams





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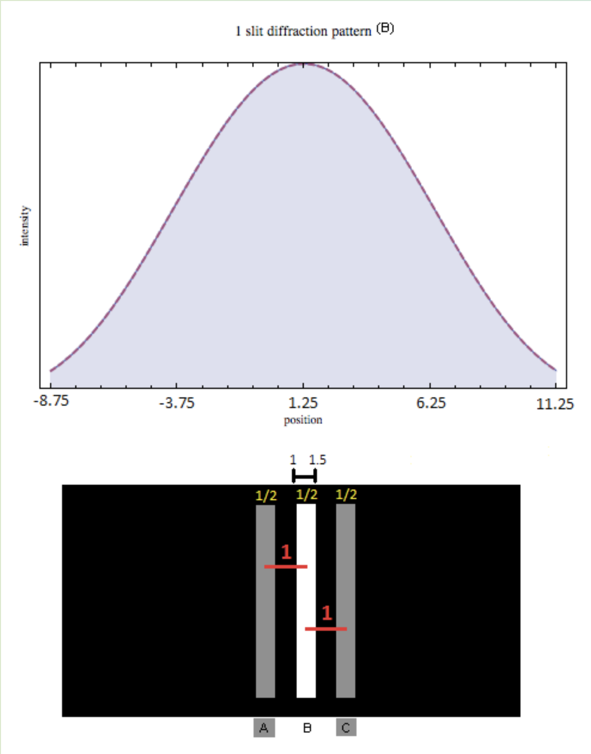
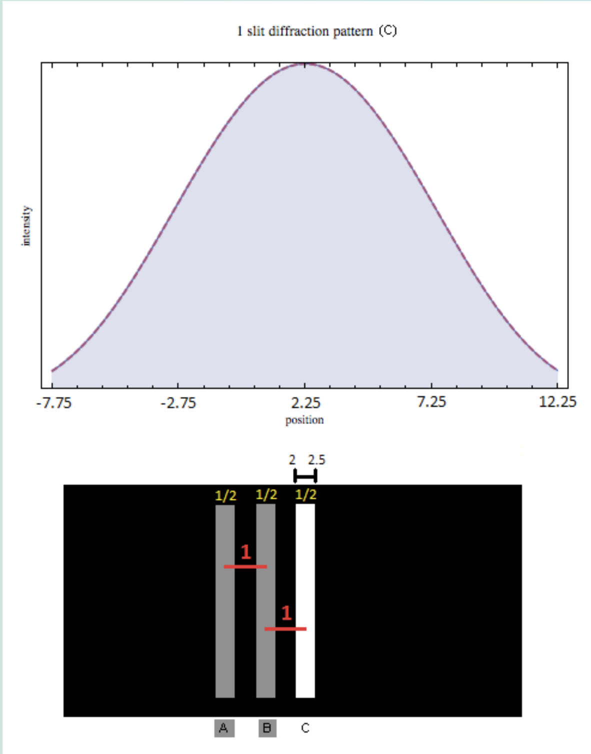
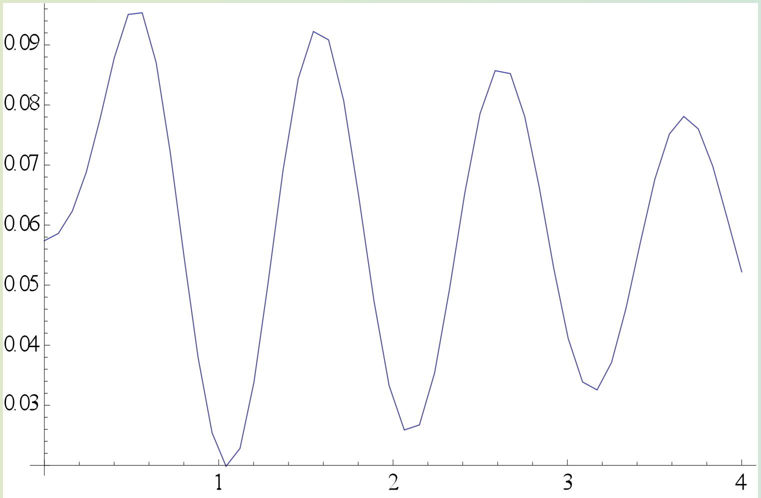


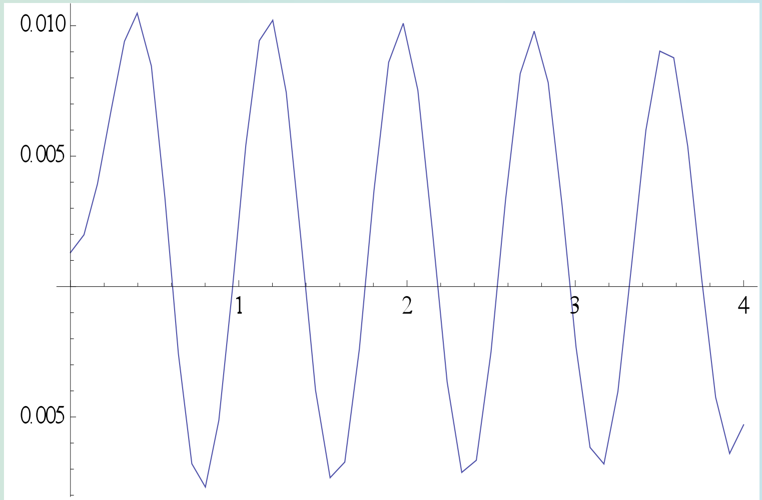


Error Analysis

Plot[{ERs[-4.75,-3.75,s]},{s,0,4}]



Plot[{ERs[9.5,10,s]},{s,0,4}]



Plot[{ERs[4.25,5.25,s]},{s,0,4}]