Computing the Equivariant K-Theory Class of the Subvariety of Triality-Symmetric Maps: Part II

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Introduction

- Based on Dave Anderson's paper: *Degeneracy of triality-symmetric morphisms* (2009)

  http://front.math.ucdavis.edu/0901.1347

- Trying to find a more general formula for a certain type of space

- Let $V = V_1 + V_2$
  - V_1, V_2 vector spaces over the field F
  - $\dim(V_1) = \dim(V_2) = 1$
  - Then dim(V) = 2
- Let Hom(V,W) be the set of all linear maps from V into W
  - Hom(V,W) is itself a vector space
  - Let End(V) = Hom(V,V)
  - V* = Hom(V,F)
  - $\mathrm{Hom}(V, W) \simeq V^* \otimes W$

- Let $H = \mathrm{Hom}(V, \mathrm{End}(V) \oplus V^*)$
  - dim(V) = 2 => dim(H) = 12
  - Let $\{\psi_{1,..},..,\psi_{12}\}$ be a basis for H
- For $\phi \in H, \phi = (\phi_{1,} \phi_2)$ where
  - $\phi_1 : V \rightarrow \mathrm{End}(V)$
    - $\phi_1 = a_1 \psi_1 + \ldots + a_8 \psi_8$
  - $\phi_2 : V \rightarrow V^*$
    - $\phi_2 = a_9 \psi_9 + \ldots + a_{12} \psi_{12}$

Goal

- Let $\Omega_r = \{ \phi \in H \mid \phi \text{ is triality - symmetric}, \operatorname{rank}(\phi) \leqslant r \}$
- Want to place conditions on the a_i in order to find a basis for Ω_r
- Using these conditions, find the free resolution of the ideal of OMEGA.
- With the appropriate free resolution, compute the K-Theory class (Grothendieck class)

Triality-Symmetric Maps

- Condition 1:

$$\phi_1(x_1)(x_2)\wedge(x_3)=$$
$$\phi_1(x_2)(x_1)\wedge(x_3)=$$
$$\phi_1(x_1)(x_3)\wedge(x_2)$$

- Condition 2:

$$\phi_2=-\phi_2{}^* \quad (\text{antisymmetric})$$

Results

- Condition 1 leads to 4 conditions on the a_i involving $a_{1,.}..,a_6$ with $a_{7,}a_8$ free
- Condition 2 leads to 1 condition involving $a_{9,.}..,a_{12.}$
- $\mathrm{Rank}(\phi)=1$ implies $\phi_2=0$.

What's Next?

Generalize to higher dimensions.