DIMACS REU Presentation 2

Sam Hopkins

July 13, 2011

Note: this will be highly highly nonrigorous.

1 Computational Problems

- We will define by example. Let

$$MTN = \{x : x \text{ is a monotone sequence}\}$$
$$INCR = \{x : x \text{ is an increasing sequence}\}.$$

- *Decision problem* for these sets: given an input x, tell me whether $x \in MTN$ (or $x \in INCR$).
- Difficulty of the decision problem. Difficulty of both MTN and $INCR$ is $\mathcal{O}(n)$.
- Difficulty of one problem with respect to another. Given MTN, can solve $INCR$ in $\mathcal{O}(1)$. We write $INCR \leq MTN$, "$INCR$ reduces to MTN."

2 Randomness

- Here are two strings:

$$00000000000000000000$$
$$10011101110101001100.$$

  Sixteen coin flips give each equal probability, but one seems more "random" than the other.

- Randomness is lack of patterns. Lack of patterns $\Rightarrow$ lack of short description. How much information do I have to give you to find my string?
- A string is random if it admits no description shorter than itself. We write R for the set of random strings.

3 The Conjecture

- We are interested in the set

$$\{A : A \leq R\}.$$

  That is, what problems (that were already solvable) become easy when you get access to the random strings?
- How in the *world* does having access to randomness help you solve a problem? Unclear what information we can get out of R.
- Conjecture:

$$\{A : A \leq R\} \subseteq \text{“easy”}.$$

4 My Progress

- Get a handle on how much information an algorithm deciding some language A with respect to R is using from A by feeding that algorithm garbage answers.
- At the beginning of the summer, we knew only that there exist garbage answers for sufficiently large inputs that don't affect the operation of the algorithm.
- Showed that for arbitrary n we can find inputs x admitting n different sets of garbage answers without affecting the operation of the algorithm.