

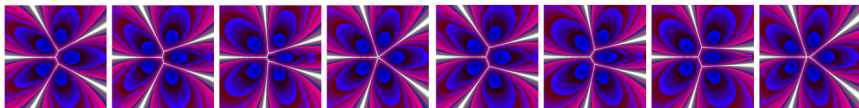
# Polynomiography

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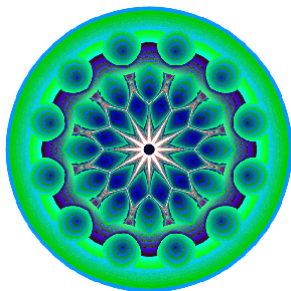
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June 8, 2012



# Introduction

What is Polynomiography?



- A game of "hide and seek" in the complex plane
- Algorithmic root approximation and visualization of complex polynomials
- The science and art of producing Polynomiographs

# Newton's Method

Newton's Iteration Function:

$$x_{n+1} = x_n - \frac{p(x_n)}{p'(x_n)}$$

where the sequence  $\{x_n\}$  converges to a particular root

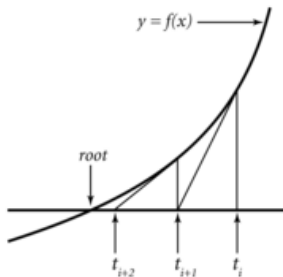
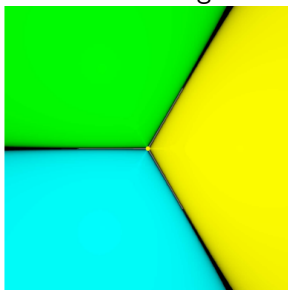


image source: <http://introcs.cs.princeton.edu/java/13flow/>

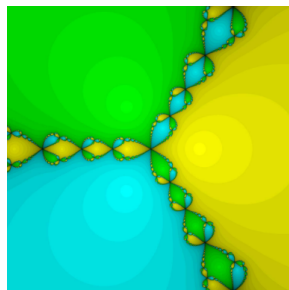
# Newton's Method: An Example

Basin of Attraction: set of all points that converge to a given root

Voronoi Diagram



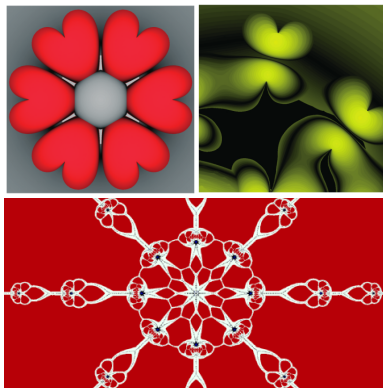
Newton's Method



$$p(x) = z^3 - 1$$

# Polynomiography

**Definition:** The art and science of visualization in the approximation of zeros of polynomials using iteration functions (Kalantari).



# Bad Polynomials

Let  $p(x) = z^3 - 2z + 2$ . Then Newton's Method is given by

$$N(z) = z - \frac{z^3 - 2z + 2}{3z^2 - 2}$$

Now we see that  $N(0) = 1$  and  $N(1) = 0$ .

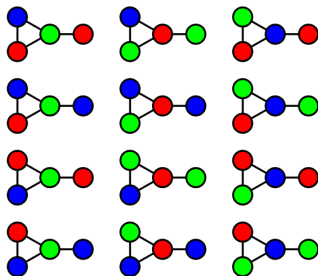
All iterative methods are susceptible to these so called 'bad' polynomials.

**Potential research:** Investigate properties of 'bad' polynomials for various iterative methods

## Other Potential Research Areas

Investigate families of polynomials the corresponding polynomiographs that arise in algebraic graph theory, such as the chromatic polynomial.

**Chromatic Polynomial:** An interpolating polynomial that counts the number of proper vertex colorings



|                     |   |   |    |    |     |
|---------------------|---|---|----|----|-----|
| Available colors    | 1 | 2 | 3  | 4  | ... |
| Number of colorings | 0 | 0 | 12 | 72 | ... |

# Sources

- Bahman Kalantari , *Polynomial Root-Finding and Polynomiography*. World Scientific, Singapore, 2009.

