Polynomiography (A second glance)

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Complex Numbers

- A complex number is a number $a+ib$ where "a" and "b" are real and i=sqrt(-1)
- $x+iy$:
  - $\cos\Theta = x/r$
  - $\sin\Theta = y/r$
    - $X=r\cos\Theta$
    - $Y=r\sin\Theta$
  - $r\cos\Theta + ir\sin\Theta$
  - $r(\cos\Theta + i\sin\Theta)$
  - $re^{i\Theta}$
- $Z_1 = x_1 + iy_1 = r_1e^{i\Theta}{}_1$
- $Z_2 = x_2 + iy_2 = r_2e^{i\Theta}{}_2$
- When two complex numbers are multiplied they reslut in rotation, with the angles summed

Roots

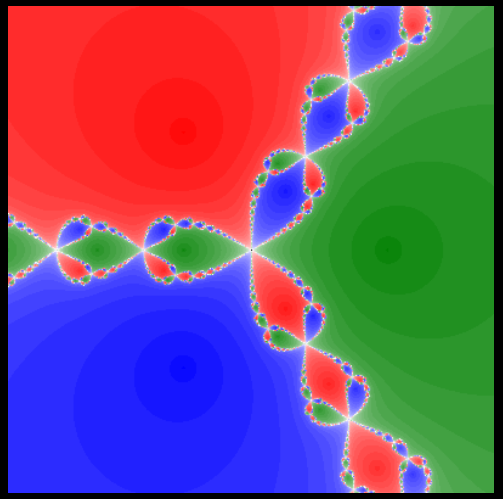
- $z^3-1=0$
  - 3 roots
  - $\Theta=1$ when it is a root
  - What other values make $\Theta=1$?
  - Take $e^{i\Theta}$ and find values of Θ that make $(e^{i\Theta})^3-1=0$
    - 2pi/3 and 4pi/3

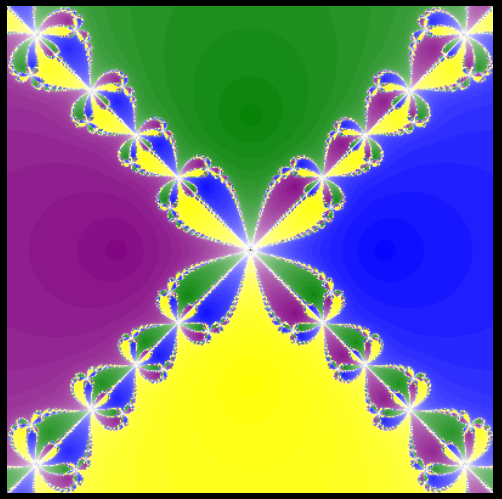
Overview of what I looked at

- Affects of:
  - Exponents
  - Coefficients
  - Multiplication
  - Symmetry/Asymmetry
- Artistic medium

Experiences

- Exponents increases the number of "branches"
  - FTA higher degree = more roots
- Changing the any coefficient changes the graph
  - Changes the roots
- Symmetry is created whenever there are real coefficient
  - Conjugate of a root is flipped over the x-axis as another root

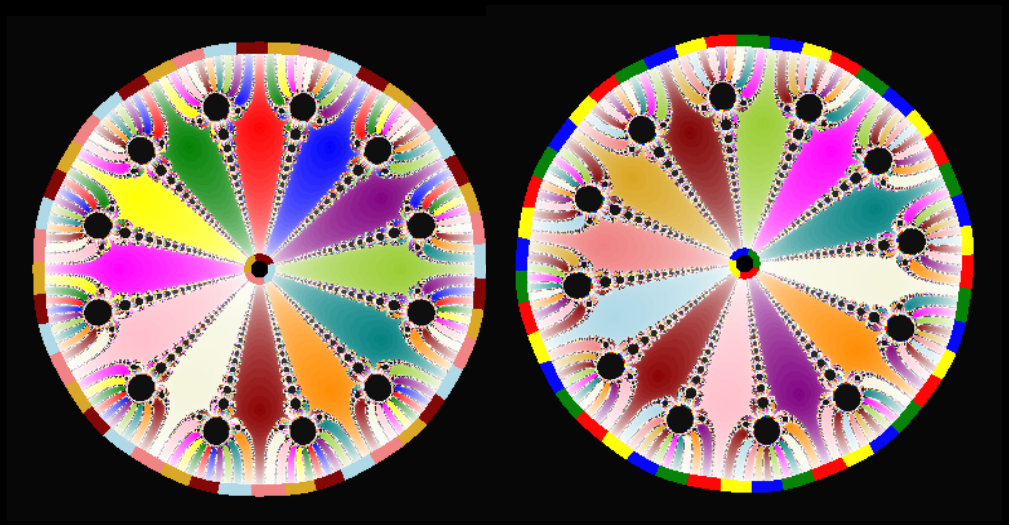




The affect of i

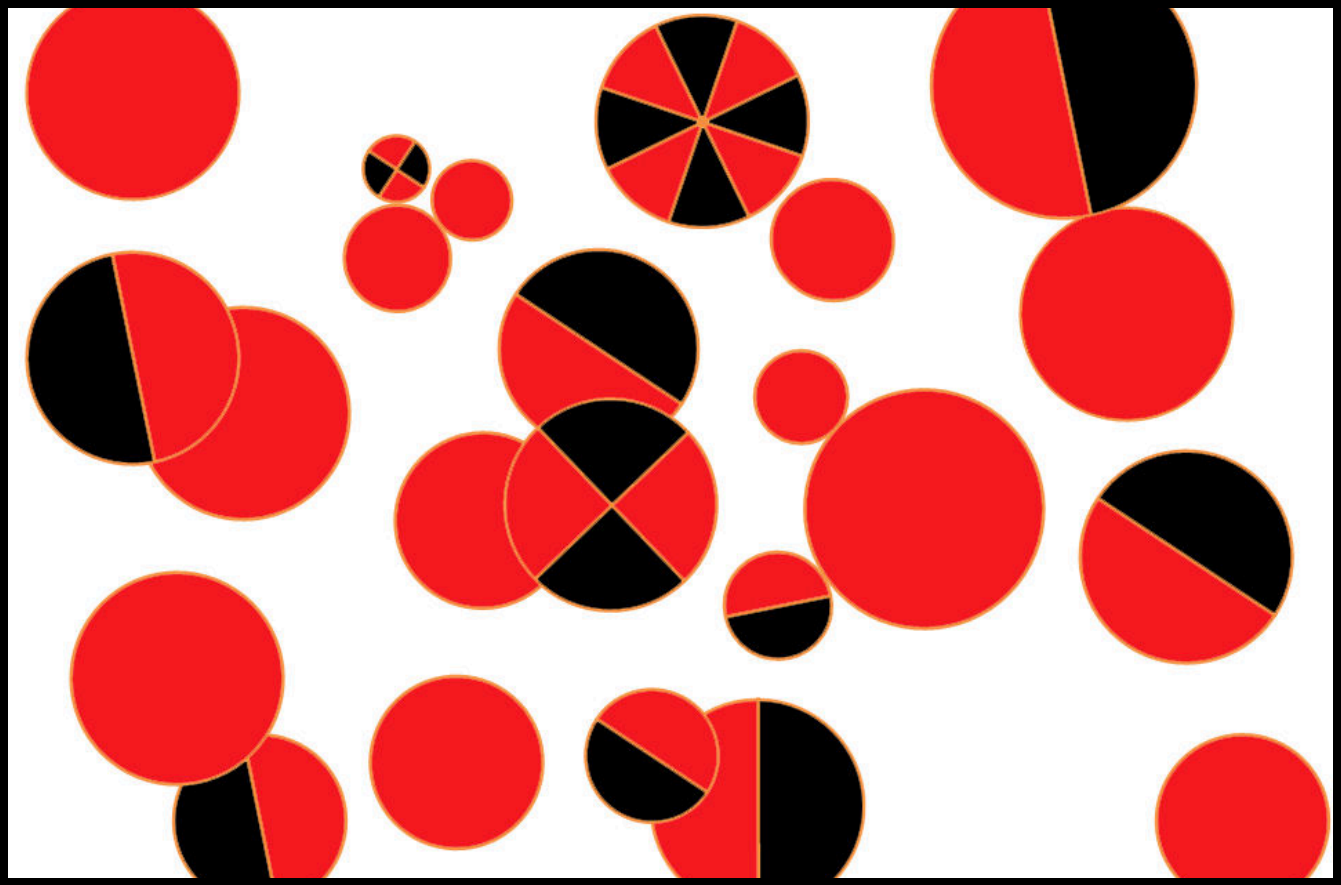
- $Z_1 = r_1 e^{i\Theta_1}$
- $Z_2 = r_2 e^{i\Theta_2}$
- $Z_1 Z_2 = r_1 r_2 (e^{i(\Theta_1 + \Theta_2)})$
- $(\Theta_1 + \Theta_2)$

- Sqrt(-1)
- Multiplying a complex number by i rotates the graph 90°



Polynomial Modulus

Theorem: Let $p(z)$ be a nonconstant polynomial. If $p(z_0) = 0$, then every direction at z_0 is an ascent direction for $|p(z_0)|$. If $p(z_0) \neq 0$, then the cones of ascent and descent directions at z_0 partition in the unit disc centered at z_0 into alternating ascent and descent sectors of equal angle pi/k, where $k \geq 1$ is the smallest index with $p^{(k)}(z_0) \neq 0$



Circle Template

At center, which points take you up or down

For polynomial modulus, 50% up /down no matter what(nicely divided)

If $p(z) \neq 0$

If $p'(z) \neq 0$ then

The value of the first derivative of the modulus $\neq 0$ is how many slices are made

A few images I found