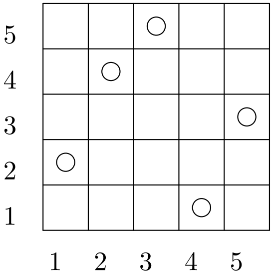
Permutations Avoiding Pattern

Presented by Ondřej Bílka

DIMACS REU 2009

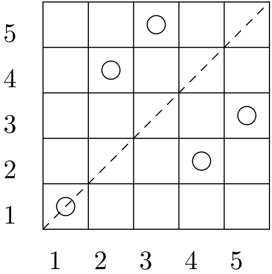
Permutations

Permutation: a mapping $\{1, 2, \ldots, n\}$ onto $\{1, 2, \ldots, n\}$.
We call the set of permutations of order n as S_n.
We can visualize permutations in this way:



Permutations

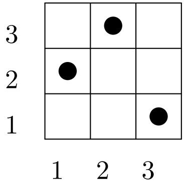
Permutation: a mapping $\{1, 2, \ldots, n\}$ onto $\{1, 2, \ldots, n\}$.
We call the set of permutations of order n as S_n.
We can visualize permutations in this way:



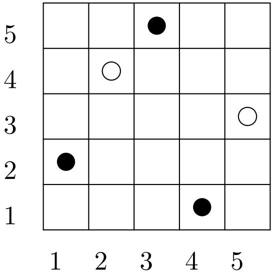
We call permutation τ *involution* if $\tau = \tau^{-1}$
We call set of involutions of order n as I_n.

Included pattern

Permutation σ contains pattern τ if we can by removing rows and collumns obtain permutation τ.
The pattern



is contained in the following permutation



Wilf's equivalence

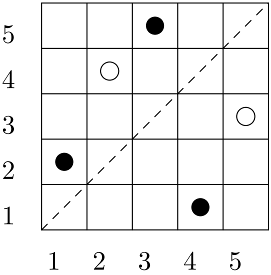
$S_n(\tau) = \{\sigma | \sigma \in S_n, \sigma \text{ does not contain } \tau\}$.
$I_n(\tau) = \{\sigma | \sigma \in I_n, \sigma \text{ does not contain } \tau\}$.

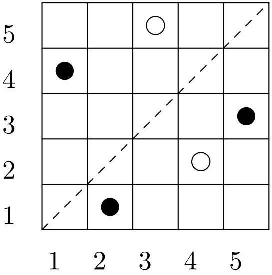
Wilf's equivalence

$S_n(\tau) = \{\sigma | \sigma \in S_n, \sigma \text{ does not contain } \tau\}$.
$I_n(\tau) = \{\sigma | \sigma \in I_n, \sigma \text{ does not contain } \tau\}$.
We call permutations σ, τ Wilf-equivalent for
permutations if $\forall n \ |S_n(\sigma)| = |S_n(\tau)|$ denoted by $\sigma \sim_S \tau$,
involutions if $\forall n \ |I_n(\sigma)| = |I_n(\tau)|$ denoted by $\sigma \sim_I \tau$.

Wilf's equivalence

$S_n(\tau) = \{\sigma | \sigma \in S_n, \sigma$ does not contain $\tau\}$.
$I_n(\tau) = \{\sigma | \sigma \in I_n, \sigma$ does not contain $\tau\}$.
We call permutations σ, τ Wilf-equivalent for
permutations if $\forall n \ |S_n(\sigma)| = |S_n(\tau)|$ denoted by $\sigma \sim_S \tau$,
involutions if $\forall n \ |I_n(\sigma)| = |I_n(\tau)|$ denoted by $\sigma \sim_I \tau$.
Observation: $\sigma \sim_S \sigma^{-1}$



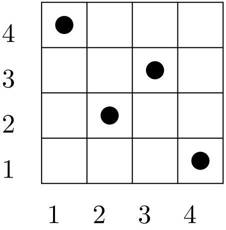


The problem

Our problem is:
Does $\sigma \sim_I \tau$ for $\sigma, \tau \in I_n$ imply that $\sigma \sim_S \tau$?

The problem

Our problem is:
Does $\sigma \sim_I \tau$ for $\sigma, \tau \in I_n$ imply that $\sigma \sim_S \tau$?
Converse is not true. For example:



and