

Ramsey theory and rado's numbers



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- Implement the algorithm in a web-friendly language (JavaScript, PHP, etc.)
- Visualizing the computation
- Prove the output (Rado numbers) with certificates.

Ramsey Theorem



Ramsey Theorem

- Given some edge-coloring of a complete graph, Ramsey's Theorem states that there exist monochromatic sub graphs (depending on how many colors are used).

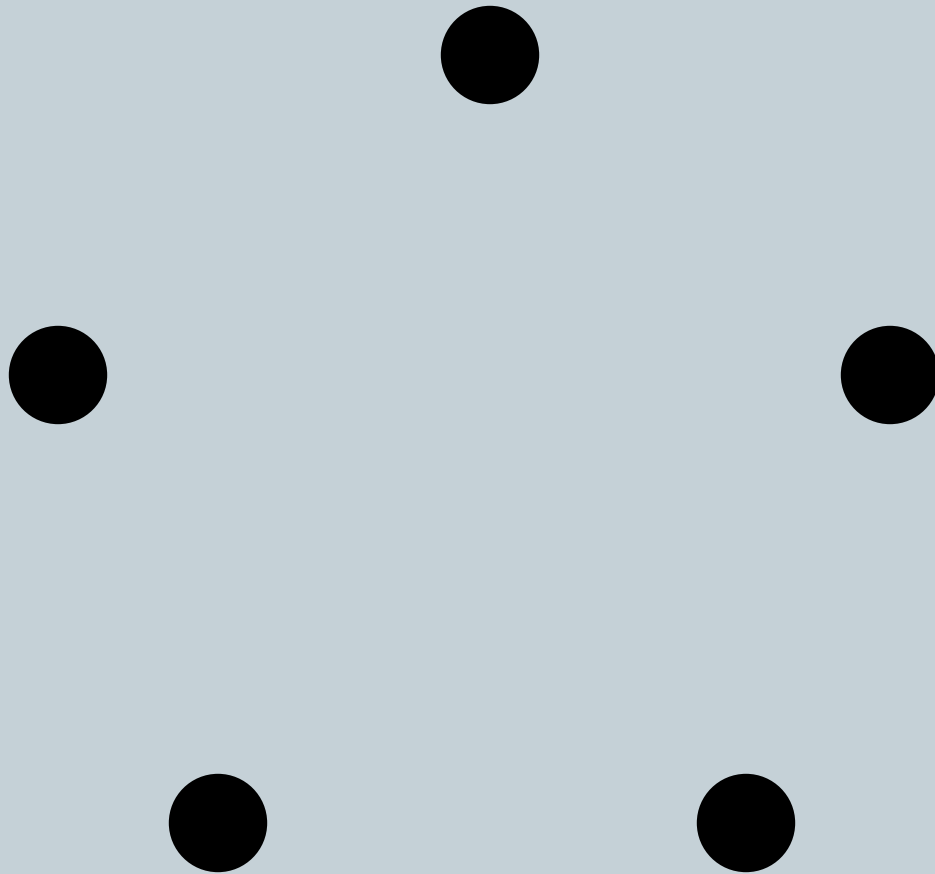
Ramsey Theorem

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- If two colors are used (red & blue), for any positive integers (r,s) , there exist a positive integer $N=R(r,s)$ where a coloring of $K[N]$ will give $K[r]$ red or $K[s]$ blue.

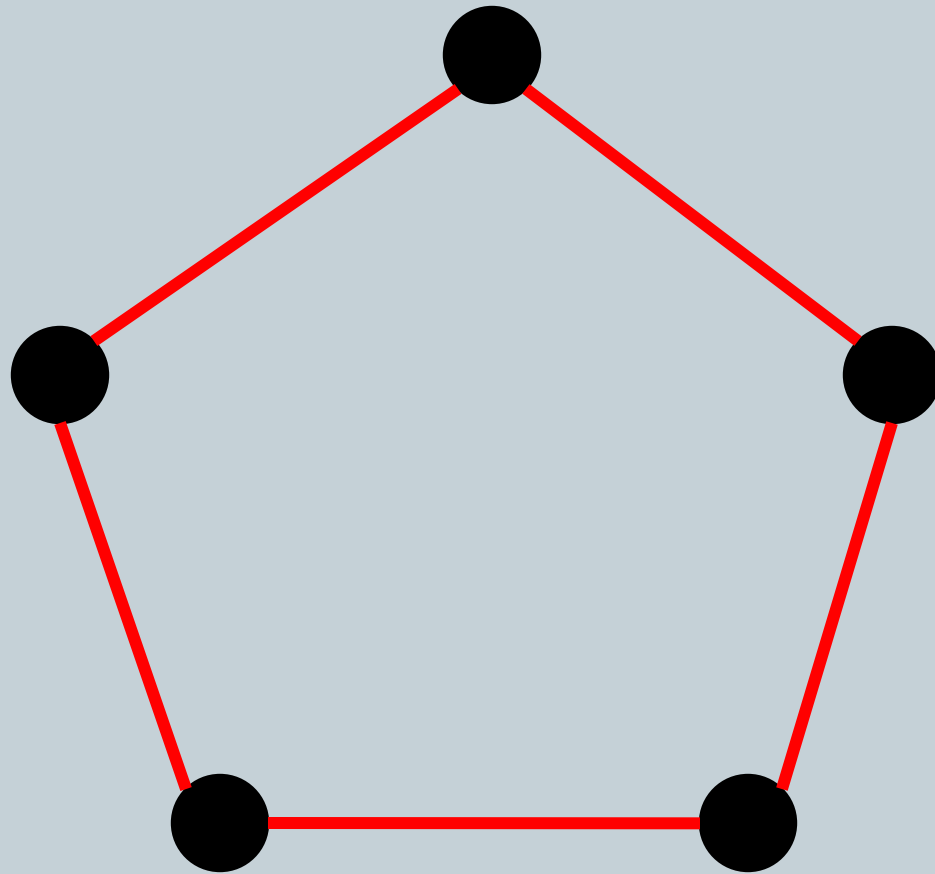
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- If two colors are used (red & blue), for any positive integers (r,s) , there exist a positive integer $N=R(r,s)$ where a coloring of $K[N]$ will give $K[r]$ red or $K[s]$ blue.
- The $R(r,s)$ is smallest integer N for which the theorem holds.

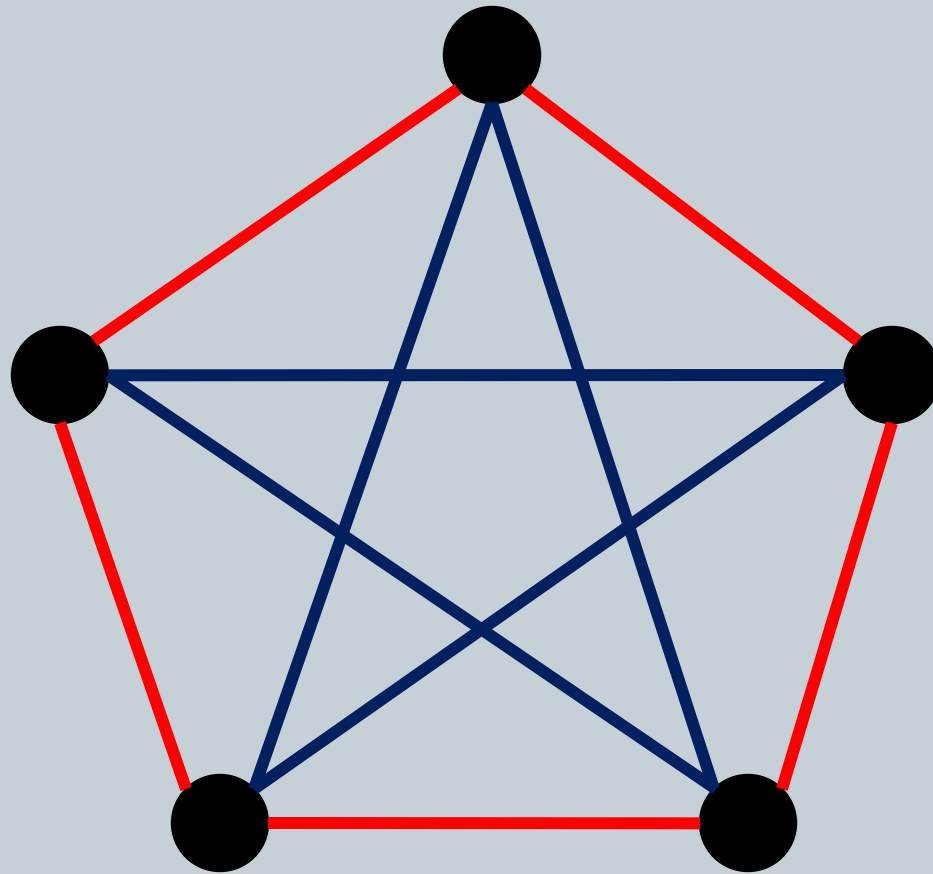
Ramsey's Theorem example



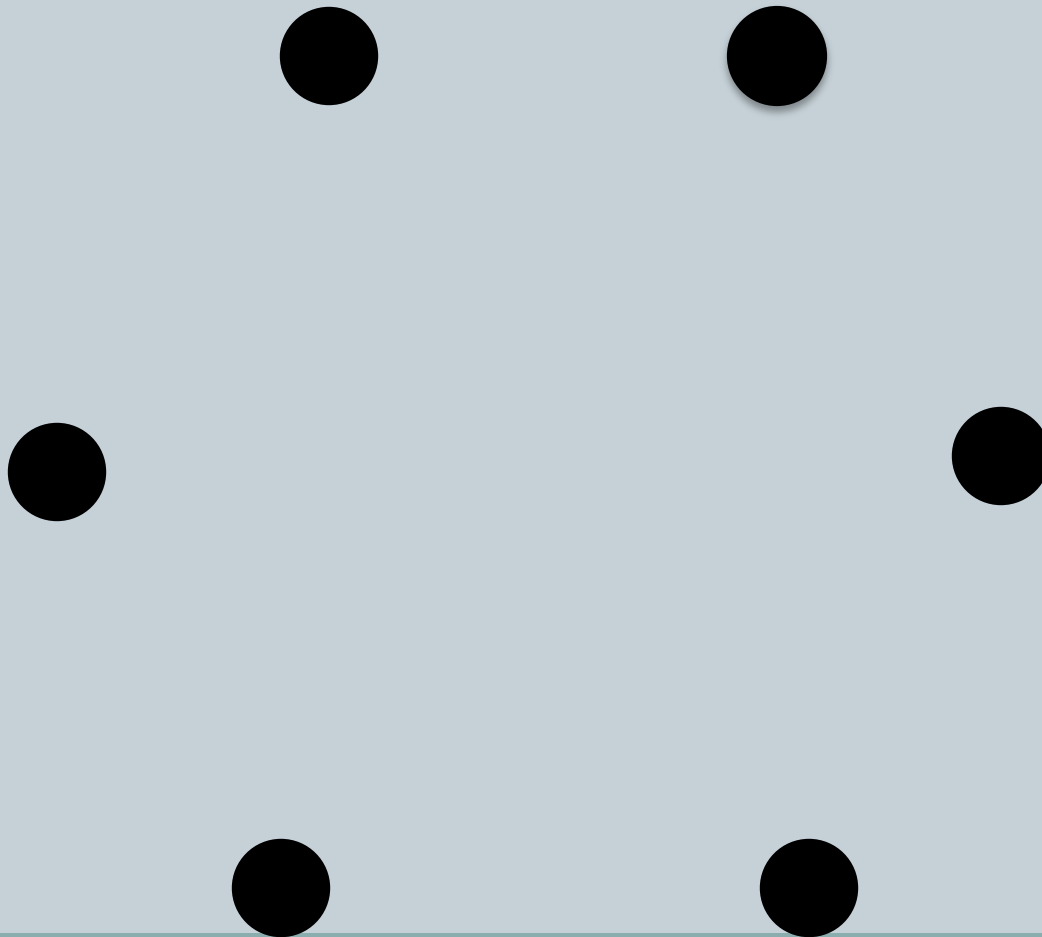
Ramsey's Theorem example



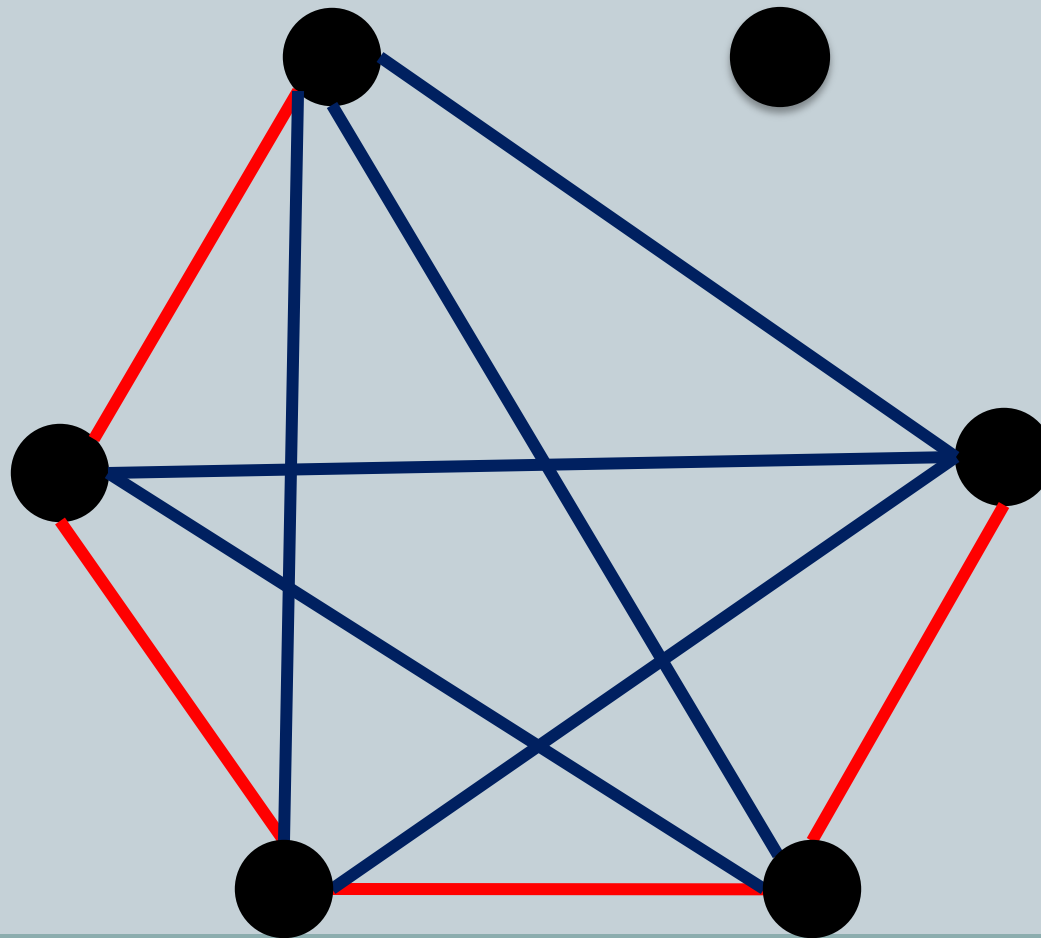
Ramsey's Theorem example



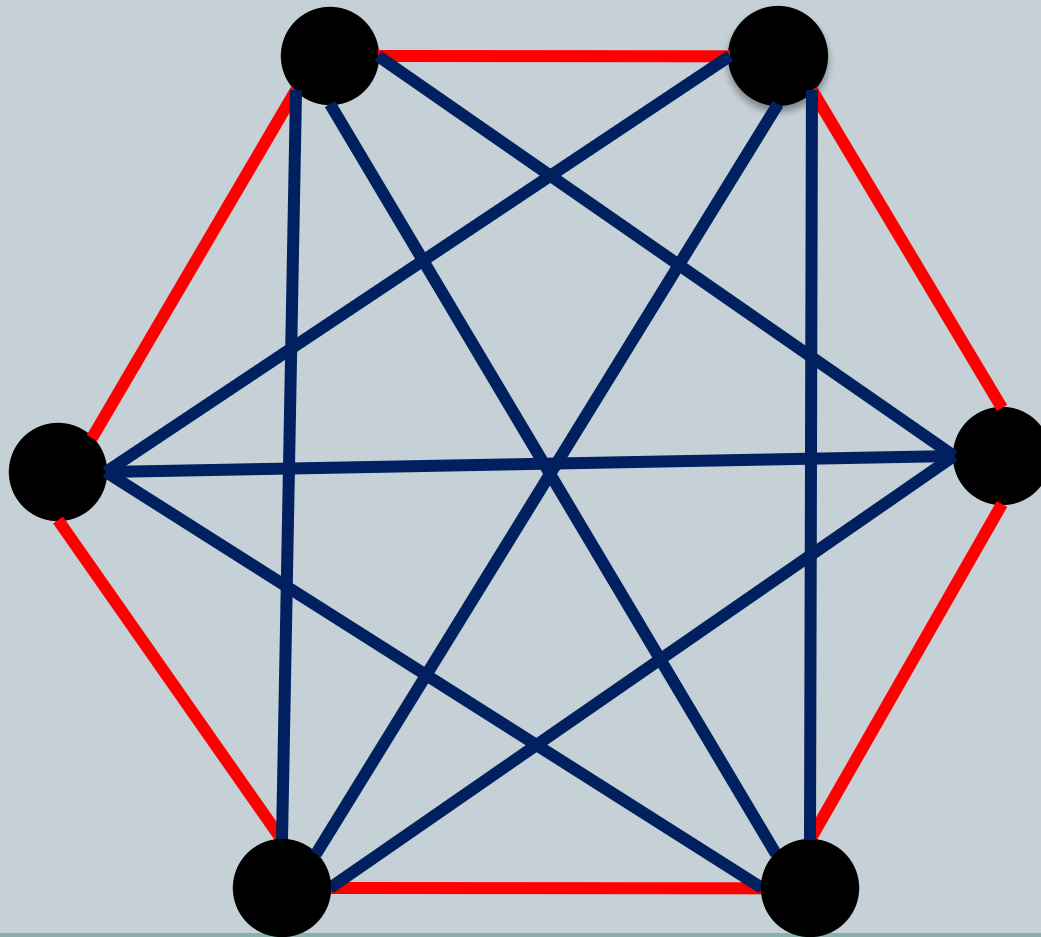
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Rado number

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- Given some equation “E” such as $x+y = z$ and some number of colors “r” ($r=2$, red and blue in this case), find the N so that there is no way to color 1 through N without a solution to the equation E of the same color.

Rado number

- Given some equation “E” such as $x+y = z$ and some number of colors “r” ($r=2$, red and blue in this case), find the N so that there is no way to color 1 through N without a solution to the equation E of the same color.
- This is analogous to $N=R(r,s)$, like $R(3,3)=6$. As showed in the graph, $K[5]$ is the smallest number you can color and avoid monochromatic $K[3]$.

Rado number example

$$x+y = z$$

Rado number example

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- $N = 1$

1

1

Rado number example

$$x+y = z$$

$$1+1 = 2$$

$$1+1 = 2$$

- $N = 1$

1

1

Rado number example

$$x+y = z$$

$$1+1 = 2$$

$$1+1 = 2$$

- $N = 1$

1

1

- $N = 2$

1 2

1 2

1 2

1 2

Rado number example

$$x+y = z$$

- $N = 1$
 - $N = 2$
- $1+1=2$ $1+1=2$
- $1+1=2$ 1 1 $1+1=2$
- ~~1 2~~ 1 2 1 2 ~~1 2~~

Rado number example

$$x+y = z$$

- $N = 1$ $1+1=2$ 1 1 $1+1=2$
- $N = 2$ $1+1=2$ $1\ 2$ $1\ 2$ $1+1=2$
- $N = 3$ $1\ 2\ 3$ $1\ 2\ 3$ $1\ 2\ 3$ $1\ 2\ 3$

Rado number example

$$x+y = z$$

- $N = 1$
 - $N = 2$
 - $N = 3$
 - $N = 4$
- $1+1=2$ $1+1=2$ $1+1=2$
- ~~1 2~~ 1 2 1 2 ~~1 2~~
- 1 2 3 1 2 3 1 2 3 1 2 3
- 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4
- 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4

Rado number example

$$x+y = z$$

$1+1=2$
 $1+1=2$

- $N = 1$

1 1
- $N = 2$

$1+1=2$
~~1 2~~ 1 2 1 2 ~~1 2~~
 $1+1=2$
- $N = 3$

1 2 3 1 2 3 1 2 3 1 2 3
- $N = 4$

~~1 2 3 4~~ ~~1 2 3 4~~ 1 2 3 4 ~~1 2 3 4~~
~~1 2 3 4~~ 1 2 3 4 ~~1 2 3 4~~ ~~1 2 3 4~~

Rado number example

$$x+y = z$$

- $N = 1$
 - $N = 2$
 - $N = 3$
 - $N = 4$
 - $N = 5$
- $1+1=2$ $1+1=2$ $1+1=2$
- $1+1=2$ $1+1=2$
- ~~1 2~~ 1 2 1 2 ~~1 2~~
- 1 2 3 1 2 3 1 2 3 1 2 3
- ~~1 2 3 4~~ ~~1 2 3 4~~ 1 2 3 4 ~~1 2 3 4~~
- ~~1 2 3 4~~ 1 2 3 4 ~~1 2 3 4~~ ~~1 2 3 4~~
- 1 2 3 4 5 1 2 3 4 5 1 2 3 4 5 1 2 3 4 5
- 1 2 3 4 5 1 2 3 4 5 1 2 3 4 5 1 2 3 4 5

Rado number example

$$x+y = z$$

$1+1=2$
 $1+1=2$

1
 1

$1+1=2$
 $1+1=2$

$N = 1$

$N = 2$
~~1 2~~
1 2
1 2
~~1 2~~

$N = 3$
1 2 3
1 2 3
1 2 3
1 2 3

$N = 4$

~~1 2 3 4~~
~~1 2 3 4~~
1 2 3 4
~~1 2 3 4~~

~~1 2 3 4~~
1 2 3 4
~~1 2 3 4~~
~~1 2 3 4~~

$N = 5$
~~1 2 3 4 5~~
~~1 2 3 4 5~~
~~1 2 3 4 5~~

~~1 2 3 4 5~~

Ultimate goal



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- Make a program that gives out the least Rado number given some equation taken from the user input.

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- If possible, it will not only work with 2 colors but with 3 or more colors.
- It will have a tree stylized output (certificate).

Tree output

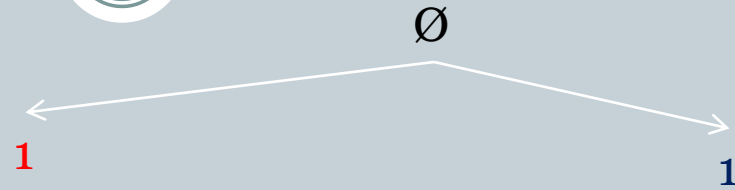


Tree output

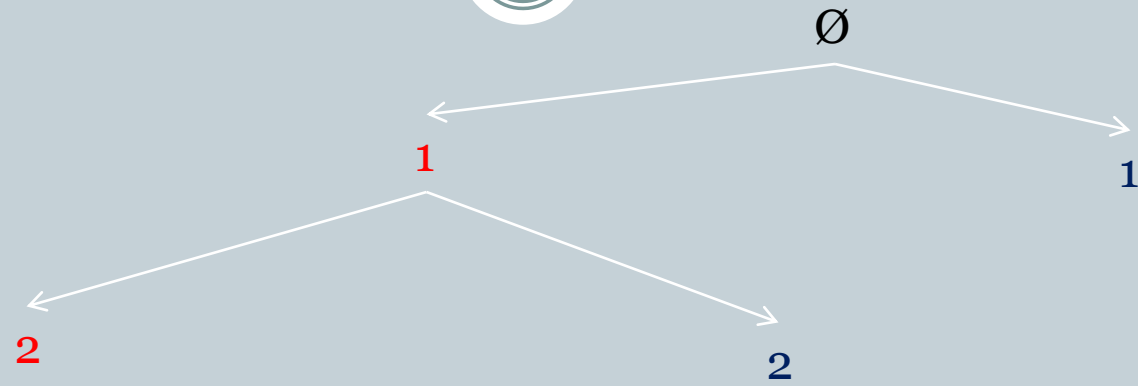


$\emptyset$

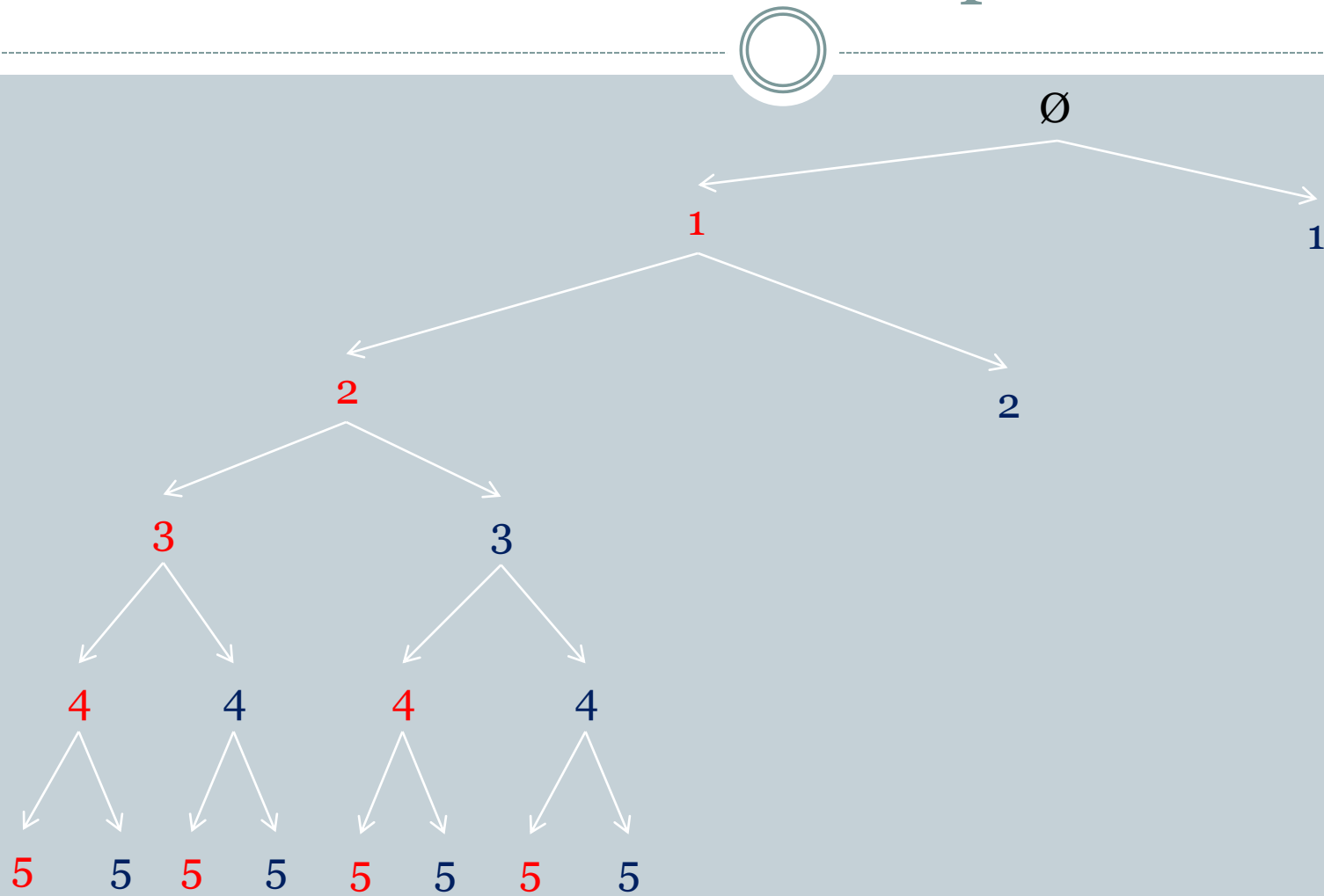
Tree output



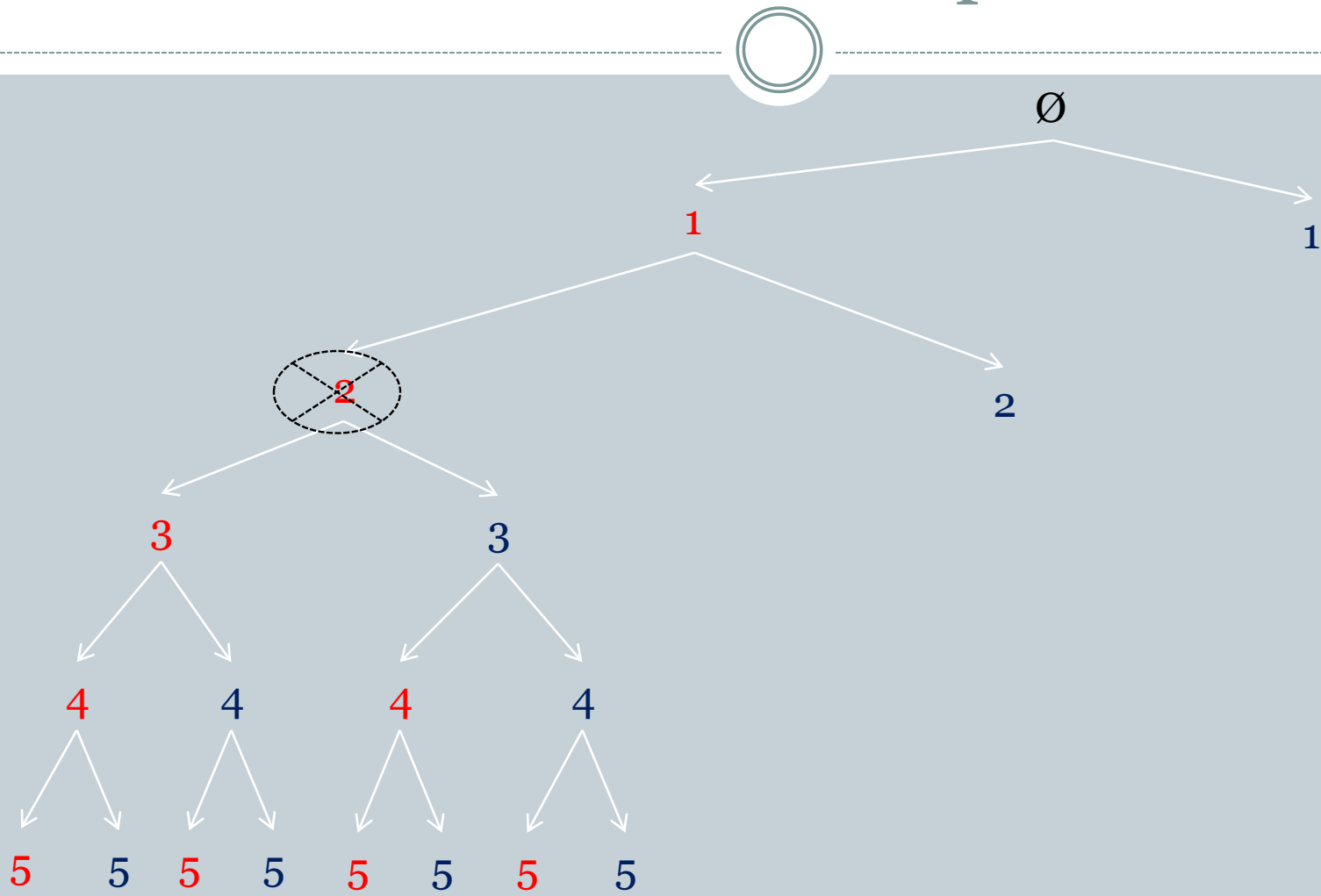
Tree output



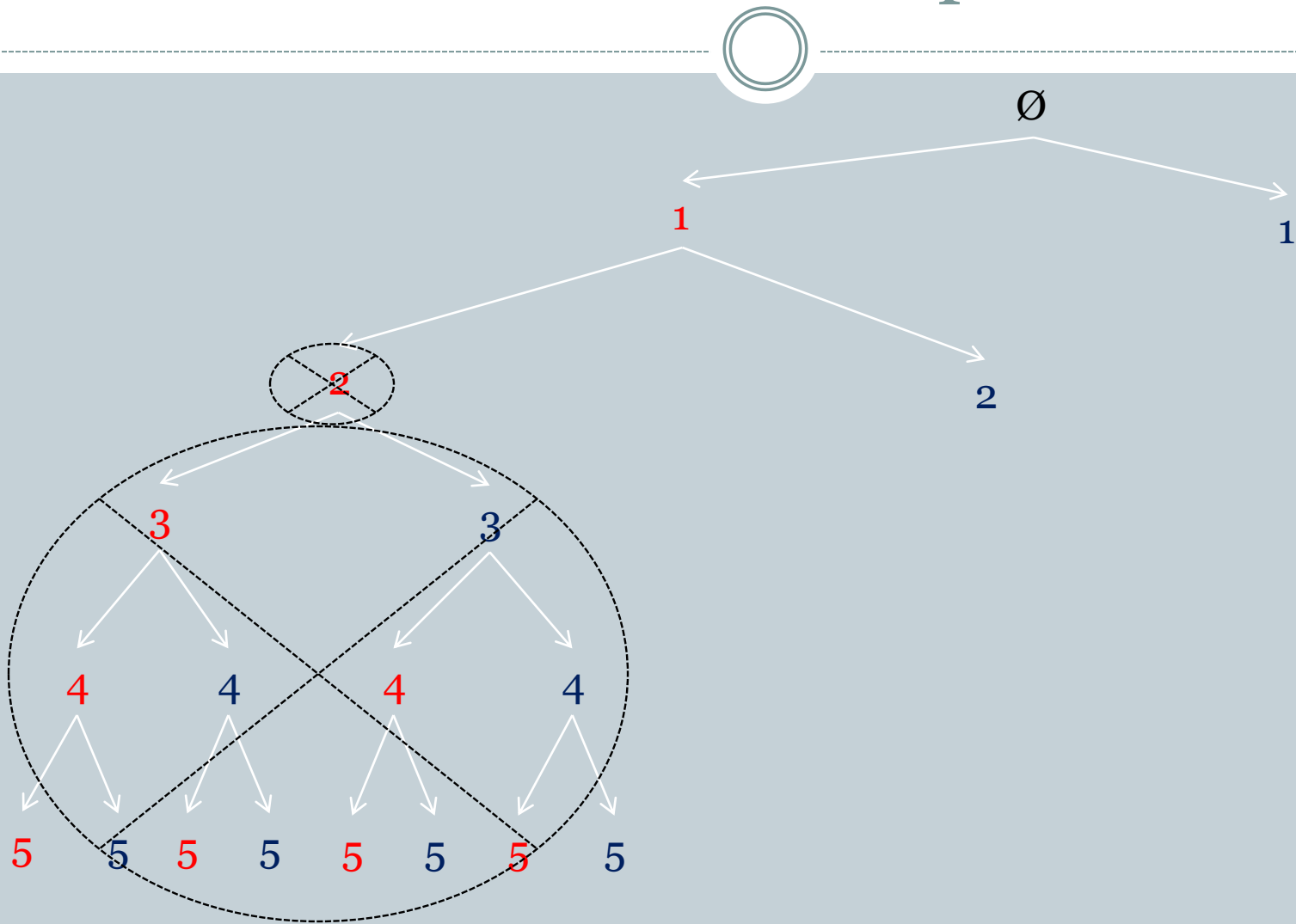
Tree output



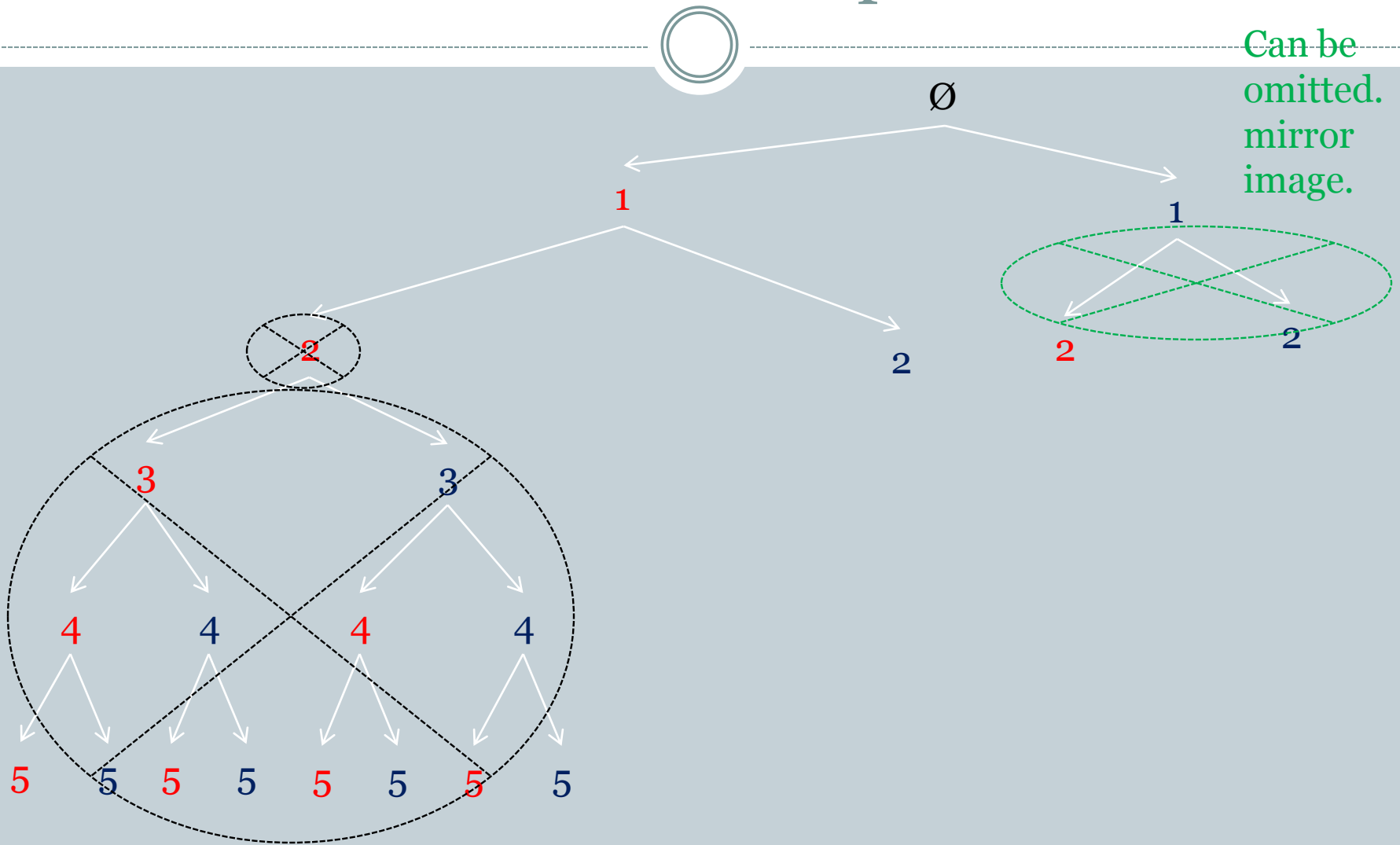
Tree output



Tree output

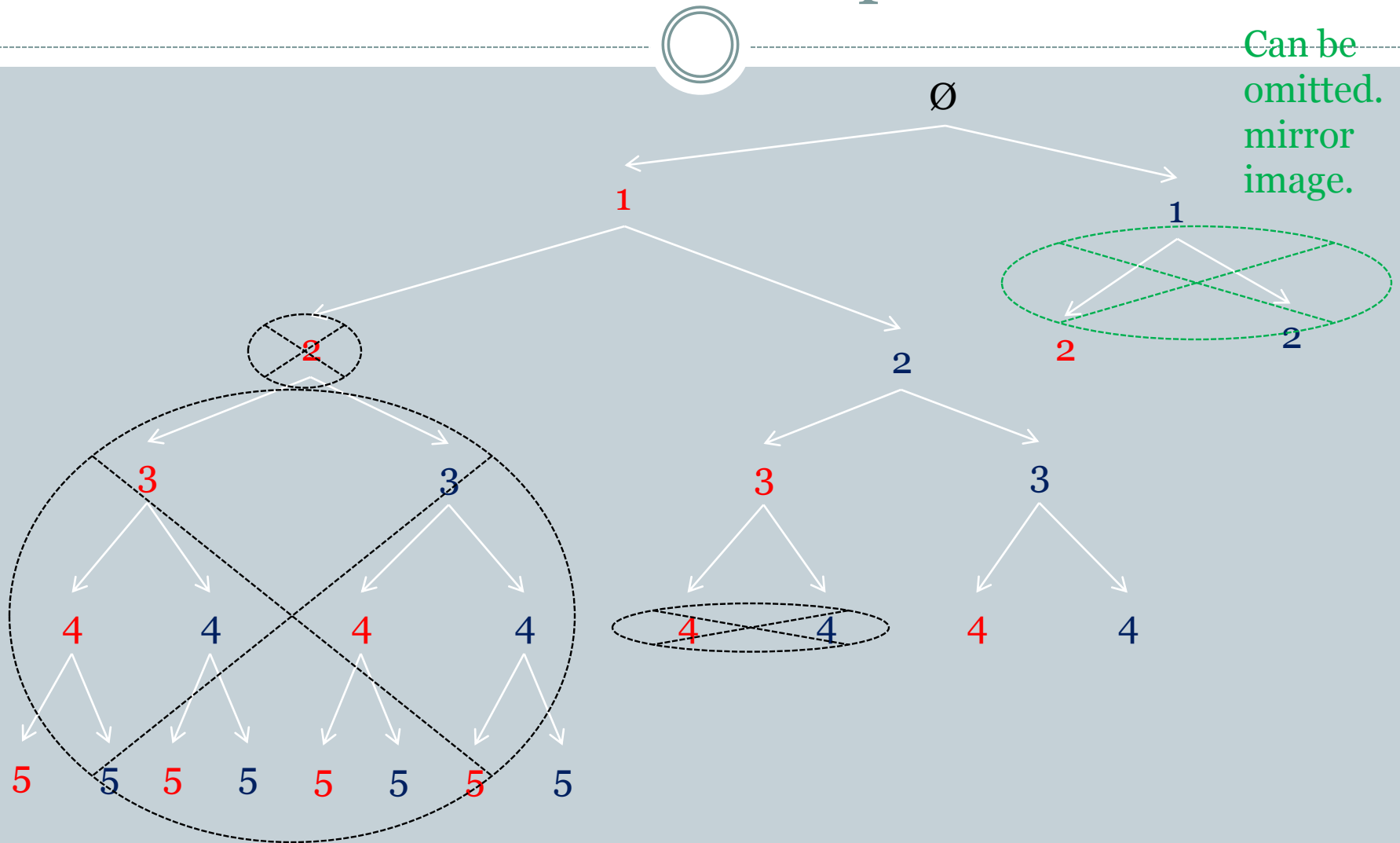


Tree output

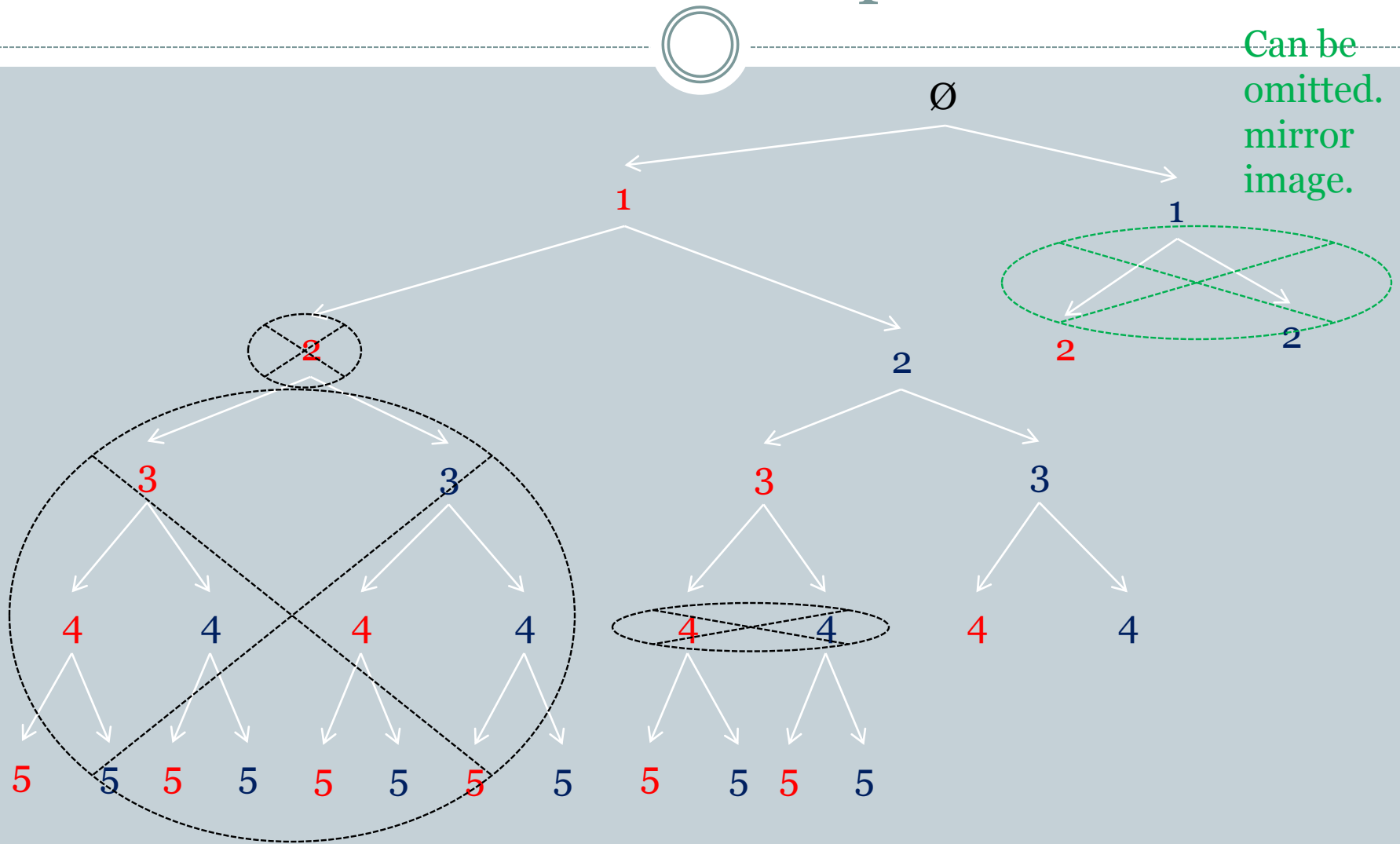




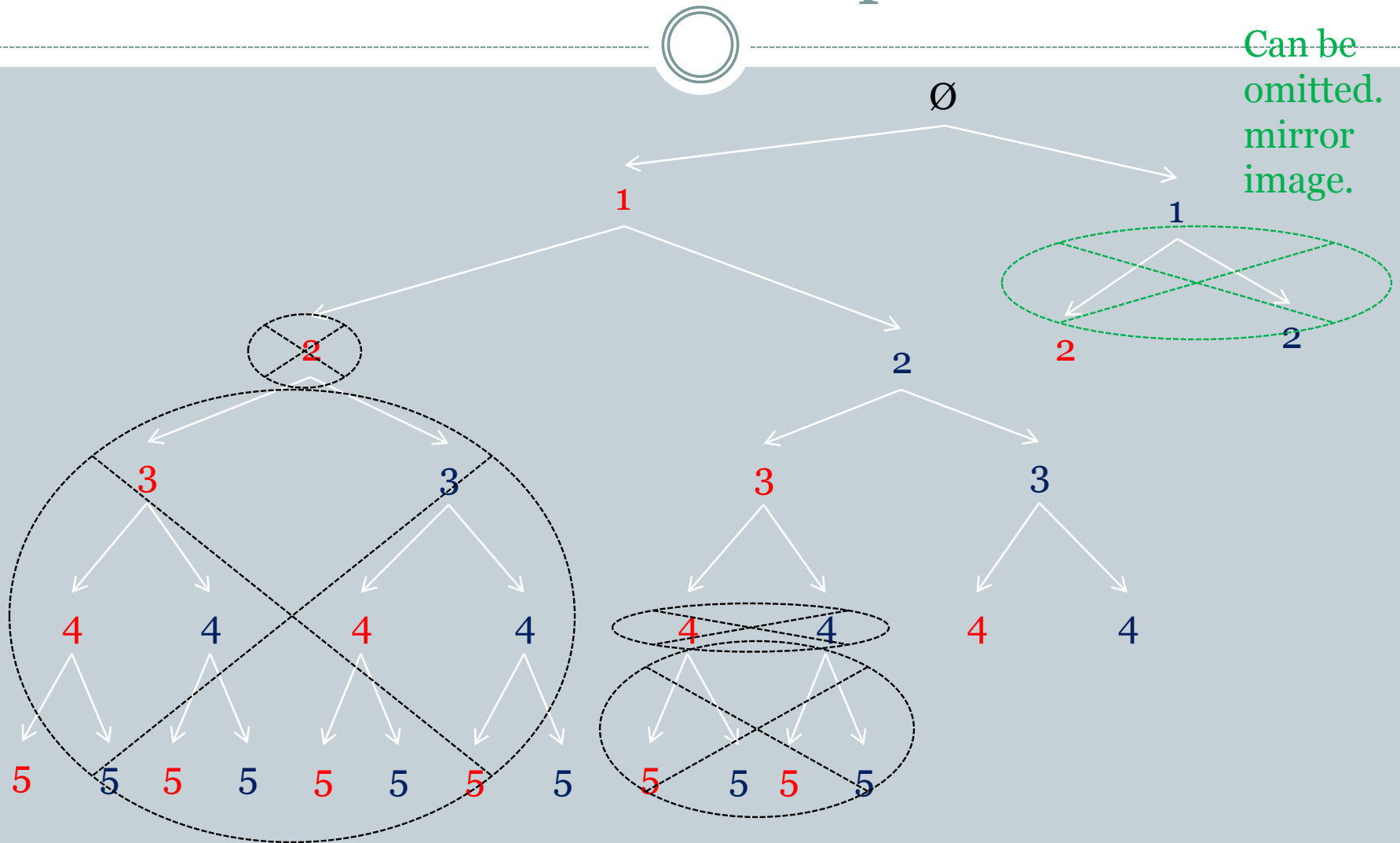
Tree output



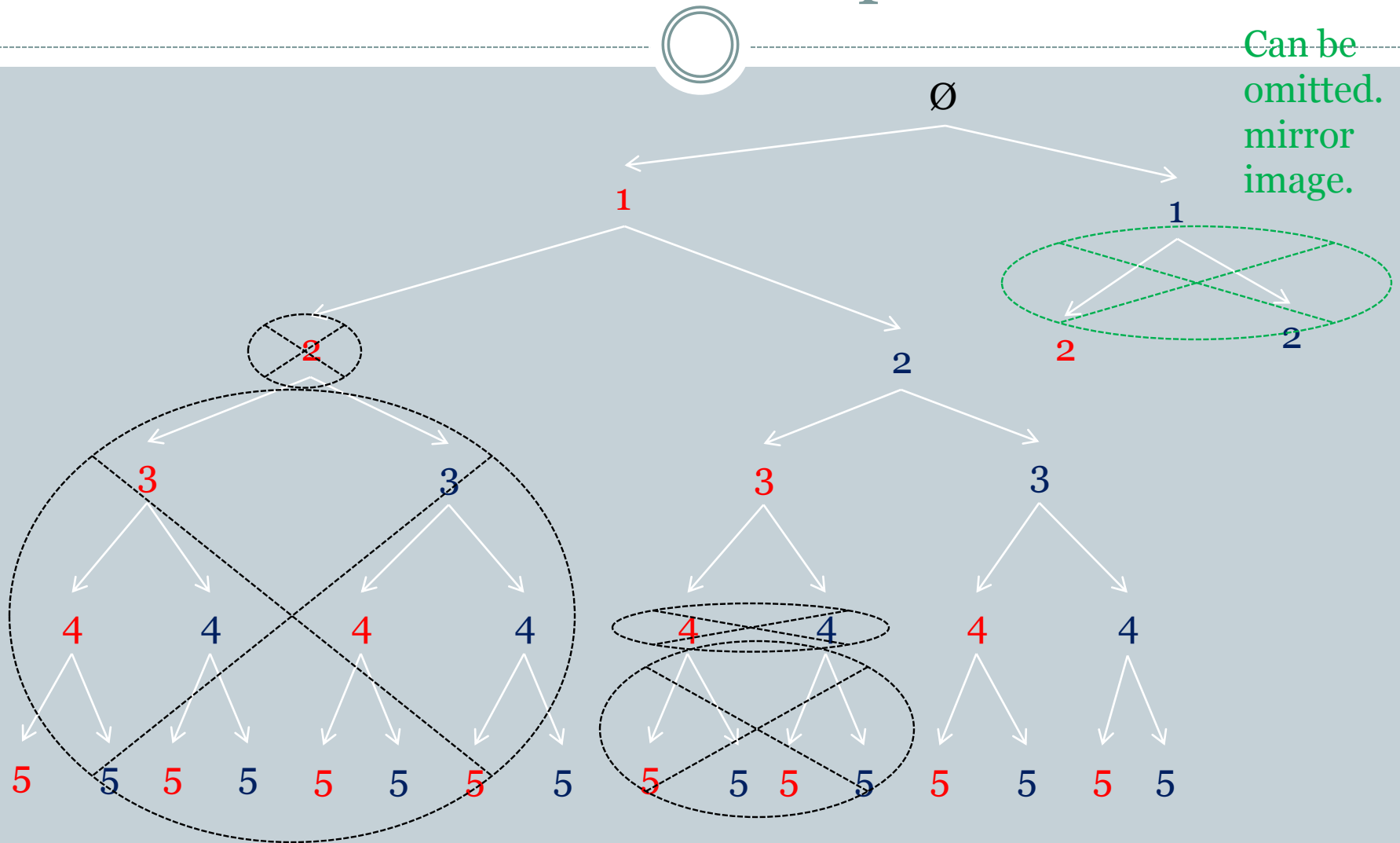
Tree output



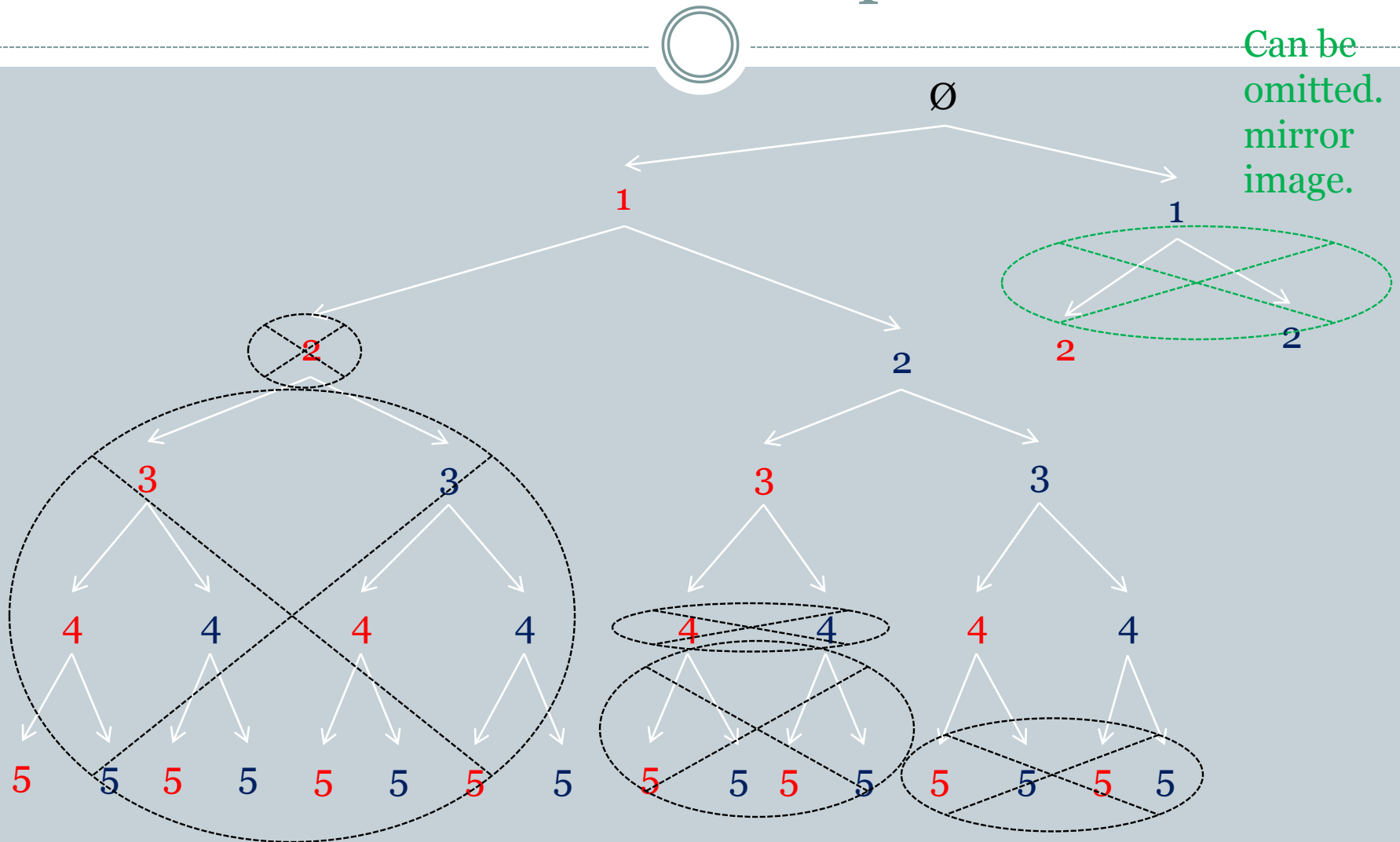
Tree output



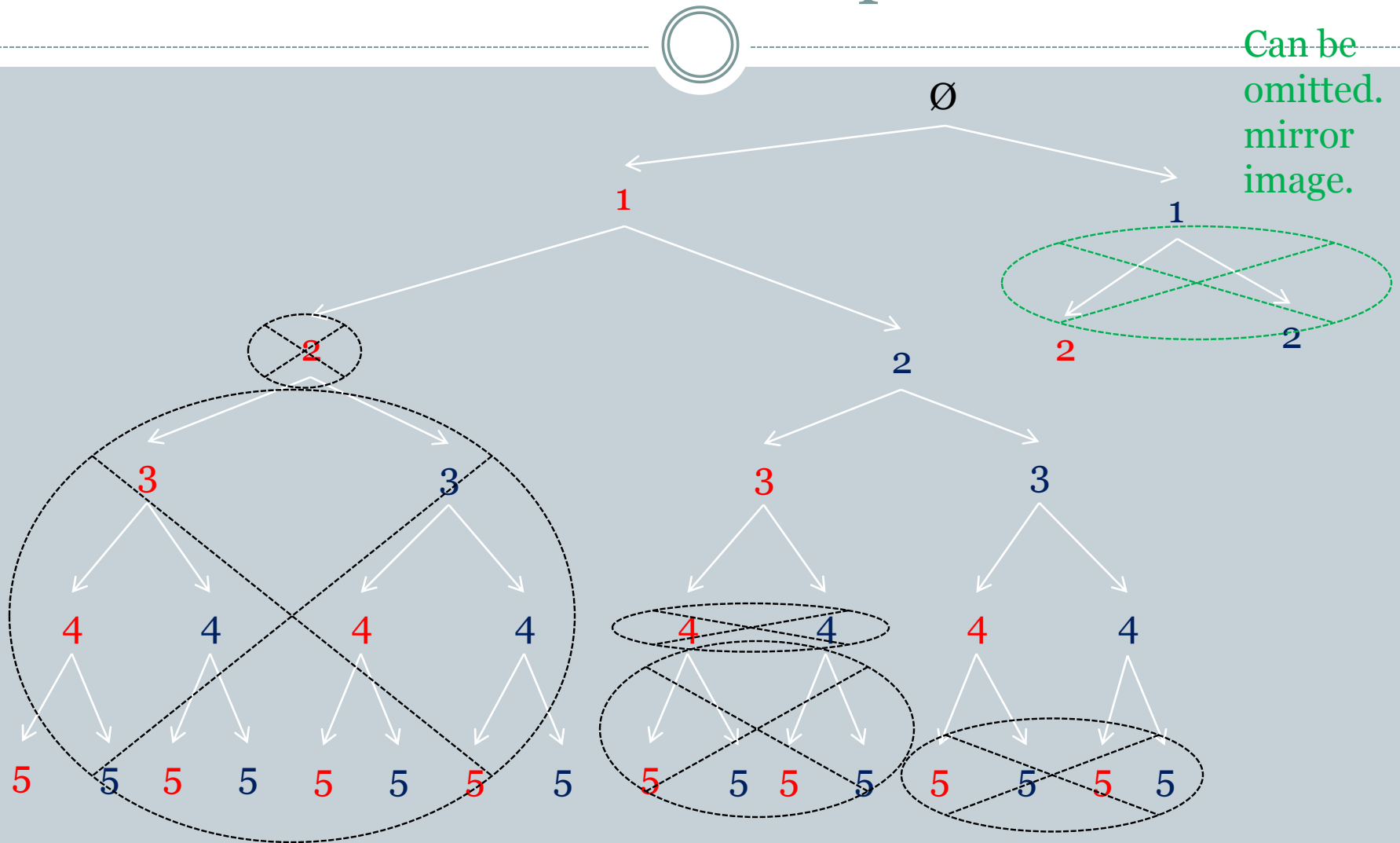
Tree output



Tree output



Tree output



Computer result



Thanks for your attention.