Ramsey theory and Rado's numbers

Adrian Galicia

Mentor: Kellen Myers

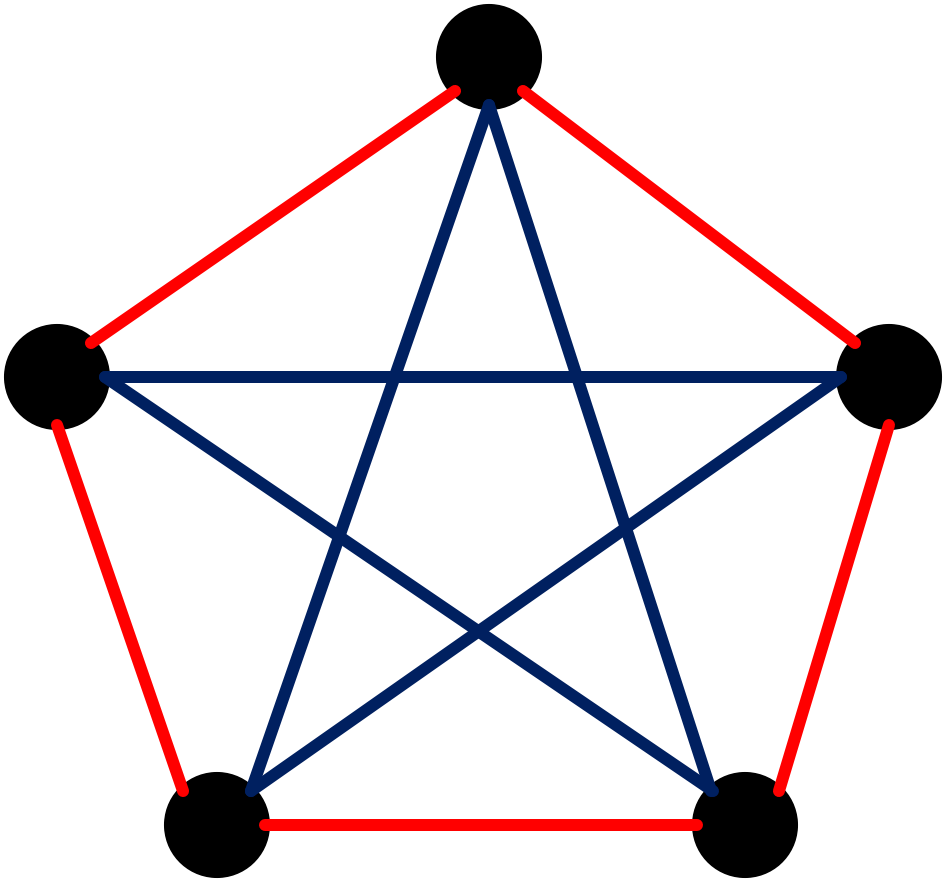
Objective

- Create an algorithm to compute Rado numbers
- Implement the algorithm in a web-friendly language (javascript, PHP, etc.)
- Visualizing the computation
- Prove the output (Rado numbers) with certificates.

Ramsey Theorem

- Given some edge-coloring of a complete graph, Ramsey's Theorem states that there exist monochromatic sub graphs (depending on how many colors are used).
- If two colors are used (red & blue), for any positive integers (r,s), there exist a positive integer N=R(r,s) where a coloring of K[N] will give K[r] red or K[s] blue.
- The R(r,s) is smallest integer N for which the theorem holds.

Ramsey's Theorem example



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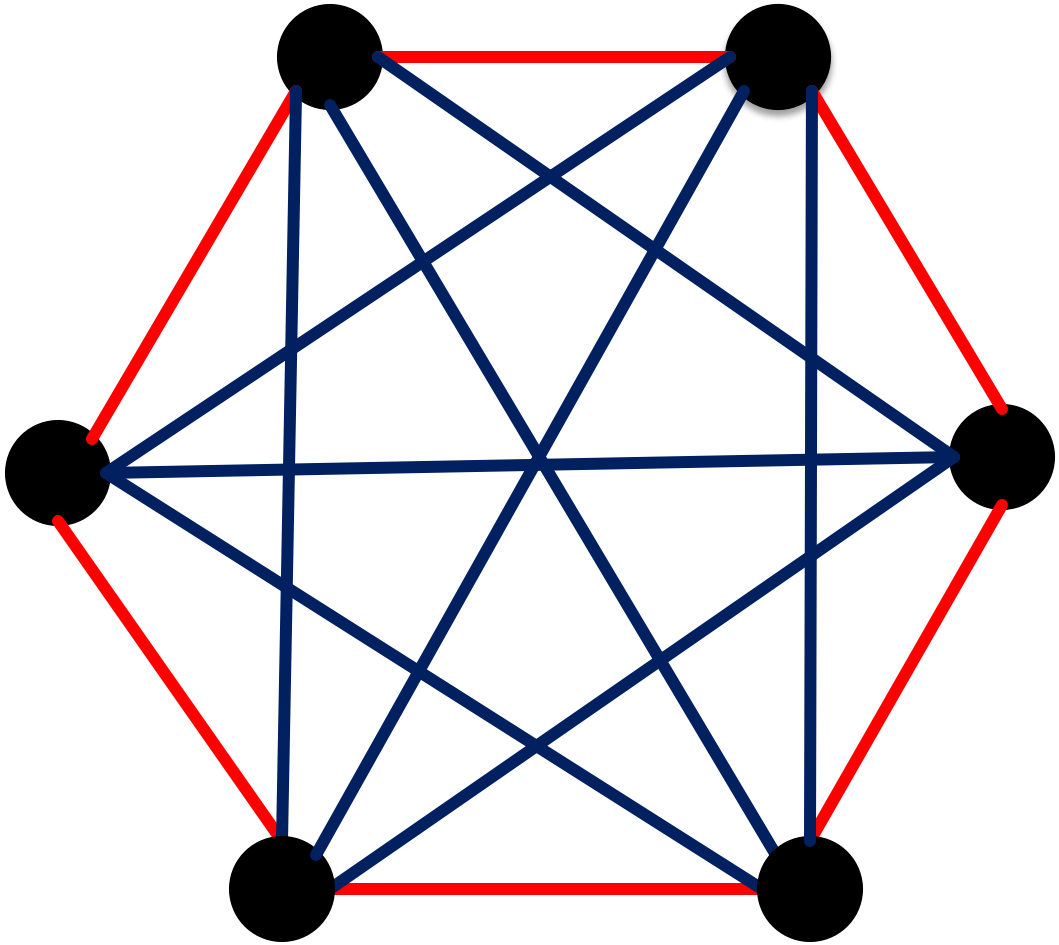
Ramsey's theorem example



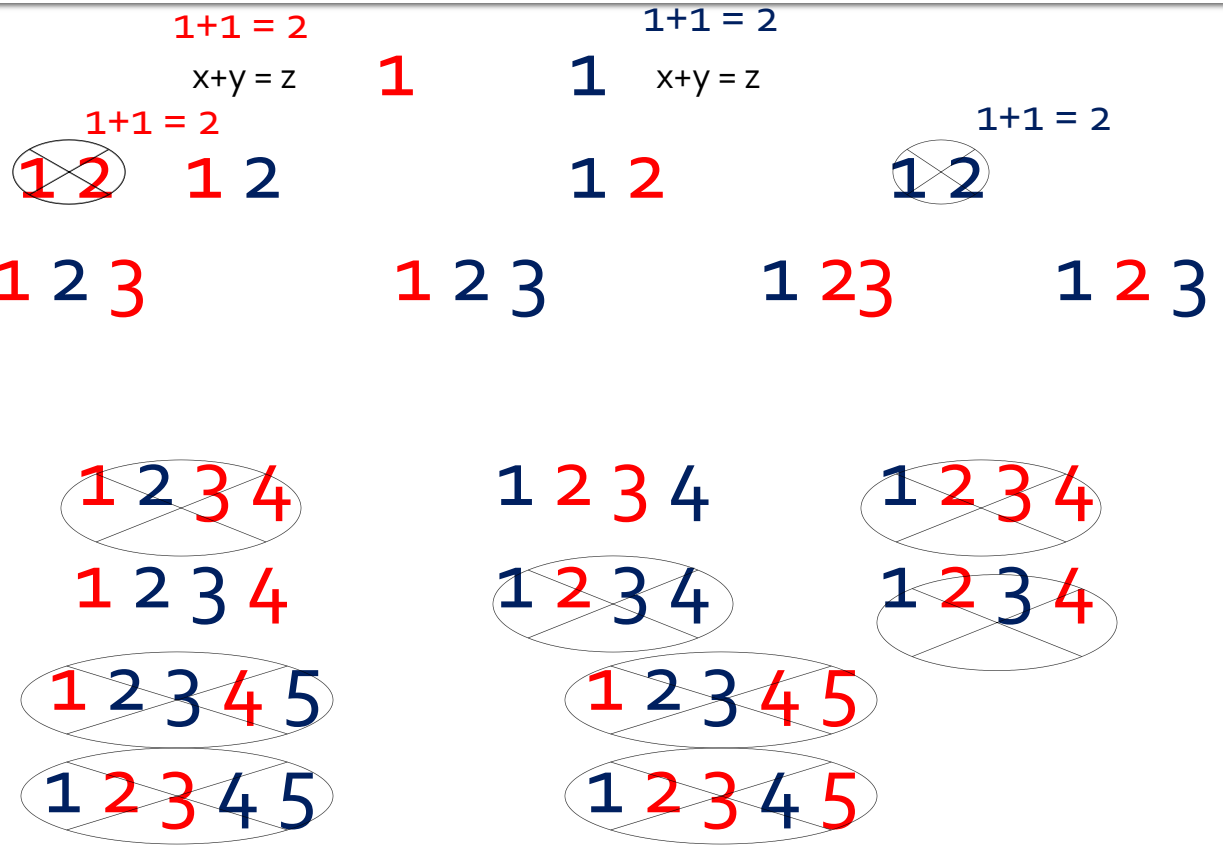
Rado number

- Given some equation "E" such as x+y = z and some number of colors "r" (r=2, red and blue in this case), find the N so that there is no way to color 1 through N without a solution to the equation E of the same color.
- This is analogous to N=R(r,s), like R(3,3)=6. As showed in the graph, K[5] is the smallest number you can color and avoid monochromatic K[3].

Rado number example

x+y = z

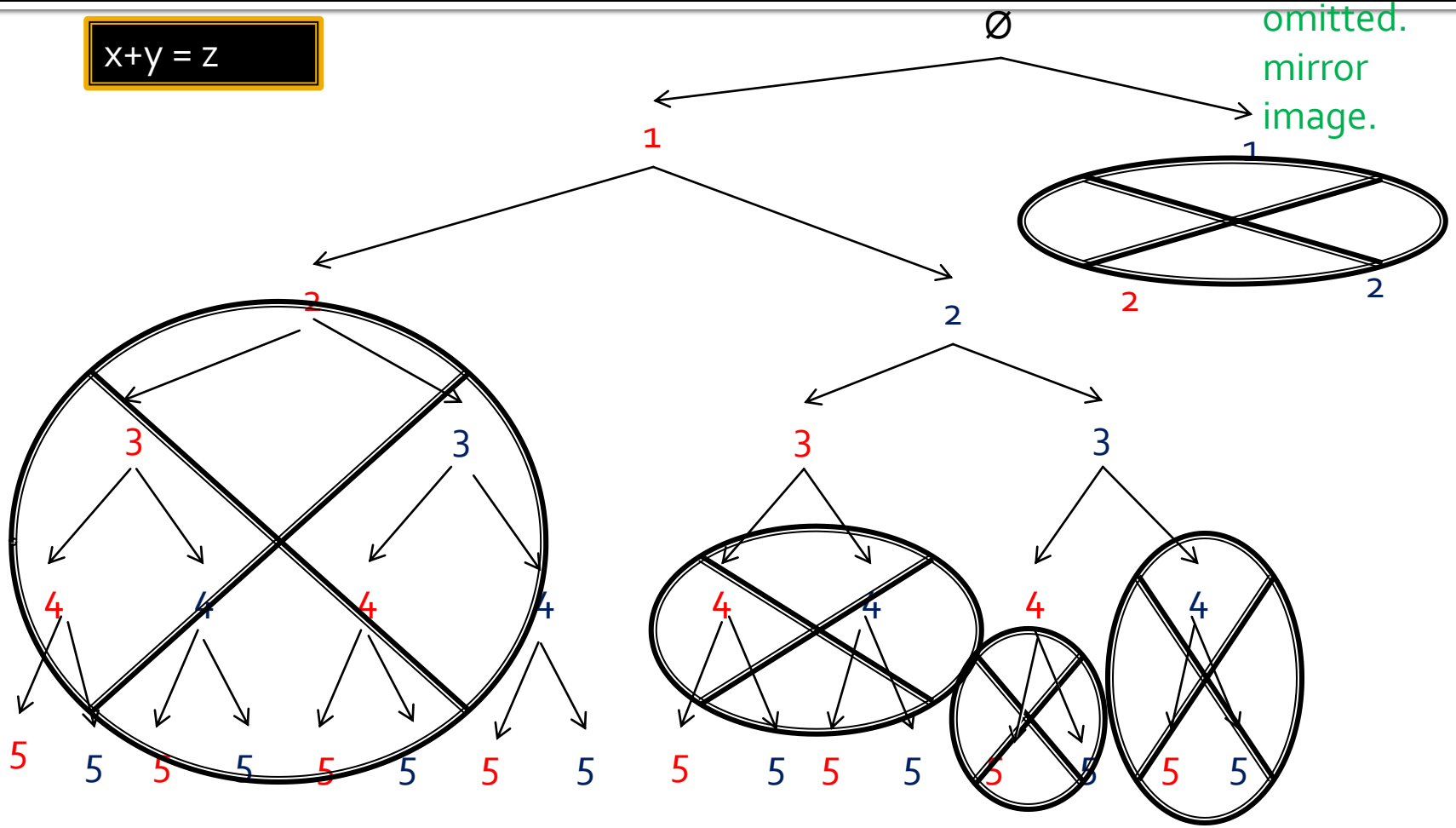
- N = 1
- N = 2
- N = 3
- N = 4
- N = 5



Tree output

x+y = z

Can be omitted. mirror image.



Ultimate goal

- Make a program that gives out the least Rado number given some equation taken from the user input.
- If possible, it will not only work with 2 colors but with 3 or more colors.
- It will have a tree stylized output (certificate).

Languages used

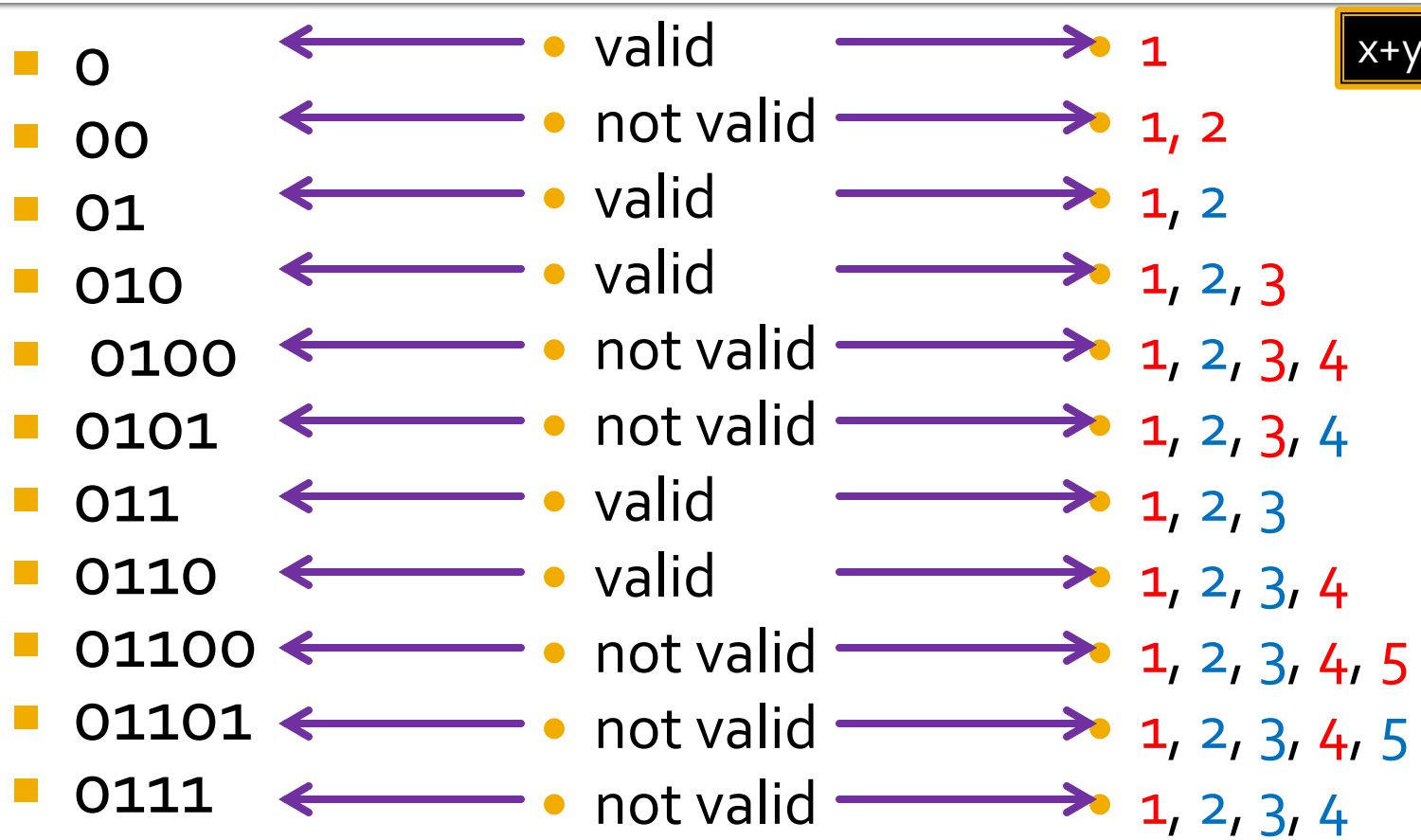
- HTML
- CSS3
- JavaScript

Demonstration

- rado_number_1.html
- Main Functions
  1. calculateN ( )
  2. checkColoring( )
  3. intoColors ()
  4. drawTree() (not finished)

How does it works(1)

x+y = z



If valid a zero is added (red number) else the last color gets replaced by the other color

How does it works (2)

Coloring : 1, 2, 3, 4, 5, 6

2x+4y = 7z

X 2(n)	Y 4(n)	X+Y combs. 4+8= 12 2+4=6	Z 7(n)
4, 8, 10	8, 16, 20	12,20,24,16,24,28, 18,26,30	14, 28, 32
2, 6, 12	4, 12, 24	6,14,26,10,18,30, 16,24,36	7, 21, 42

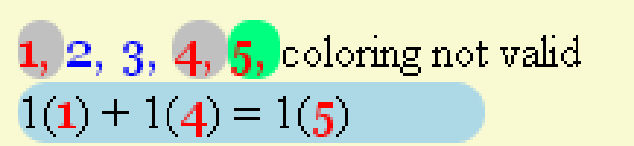
If a number in the 3rd column is in the 4th column , then that is the monochromatic solution to the equation which means that the coloring is not valid

How does it works(3)

x+y = z

- intoColors()
  - Takes the string of zeros and ones [0,1,1,0,0] and converts them into a numbers with colors

    [1, 2, 3, 4, 5] to be outputted in the screen.
  - If a coloring is not valid it highlights the values for which the coloring has a monochromatic solution for a given equation.



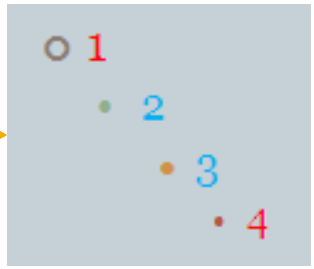
How it works (4)

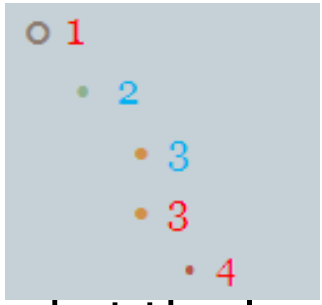
x+y = z

- drawTree( )
  - Takes the biggest non extensible colorings and orders them by length.

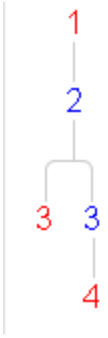
0,1,1,0
0,1,0

  - Makes a list structure of the longest one.

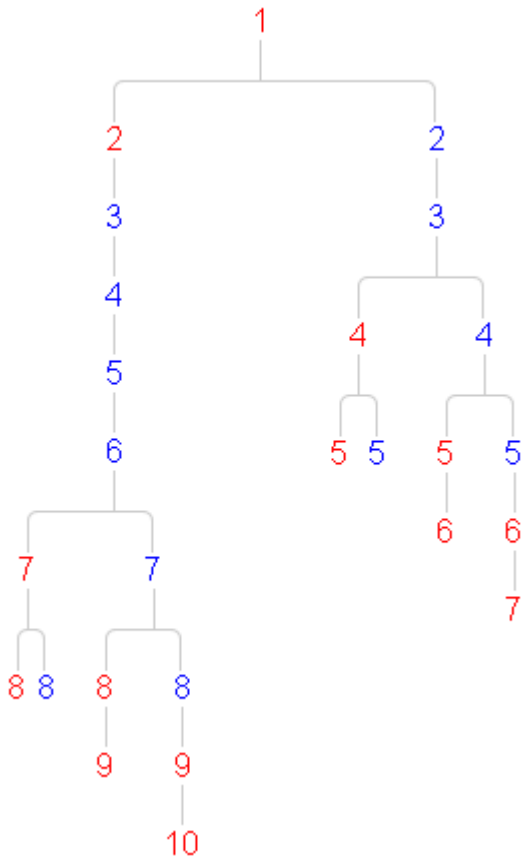




  - Then compares it to the next coloring and at the location where they differ adds a colored number
  - And then a CSS style sheet changes its appearance to an actual tree



Another tree



x+2y = z

Future Work

- Finish the tree output
- Add feature of testing a single coloring on demand.
- Test several equations
- More variables