PERFECTLY MATCHED LAYERS AND THE ART OF CLOAKING

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Outline

I. Finite-Difference Time-Domain Problems

II. Posing the Open Boundary Problem

III. Perfectly Matched Layers

IV. Current Research in Cloaking

Finite-Difference Time-Domain (FDTD)

- Consider one dimensional wave equation problem with Dirichlet boundary conditions:

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} \cdot \frac{1}{c^2} \text{ for } 0 \le t \le T, 0 \le x \le \ell$$

$$u(0; x) = f(x) \qquad \frac{\partial u(0; x)}{\partial t} = g(x)$$

$$u(t; 0) = \Psi_1(t) \qquad u(t; \ell) = \Psi_2(t).$$

- The FDTD approach is to discretize the region into a mesh of n by m cells for which we hope

$$U_{i,j} \approx u(\Delta t \cdot i; \Delta x \cdot j)$$

for $i = 0, \ldots, n-1$, $j = 0, \ldots, m-1$, $\Delta t = {}^{T}/_{n}$, and $\Delta x = {}^{\ell}/_{m}$

Finite-Difference Time-Domain (FDTD)

- From our problem, we can derive

$$U_{0,j} = f(\Delta x \cdot j),\;\; U_{i,0} = \Psi_1(\Delta t \cdot i),\;\; U_{i,m-1} = \Psi_2(\Delta t \cdot i),$$

$$U_{1,j} = g(\Delta x \cdot j)\Delta t + \frac{s}{2} \cdot f\big(\Delta x \cdot (j+1)\big) +$$

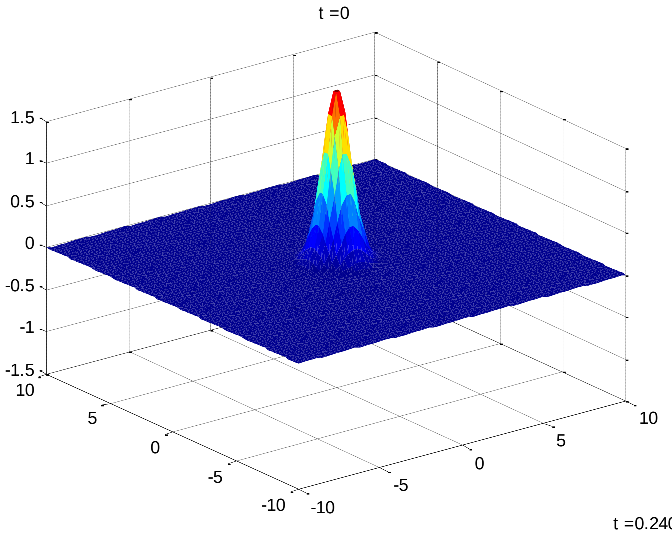
$$(1-s) \cdot f(\Delta x \cdot j) + \frac{s}{2} \cdot f\big(\Delta x \cdot (j-1)\big),$$ and

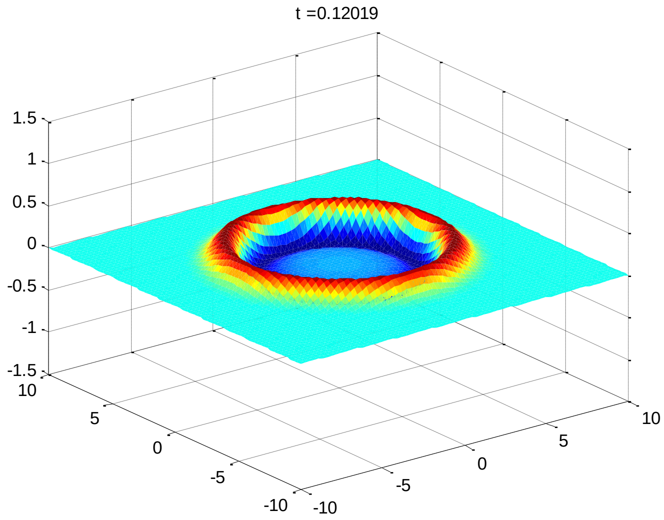
$$U_{i+1,j} = s \cdot U_{i,j+1} + 2 \cdot (1-s) \cdot U_{i,j} + s \cdot U_{i,j-1} - U_{i-1,j}$$

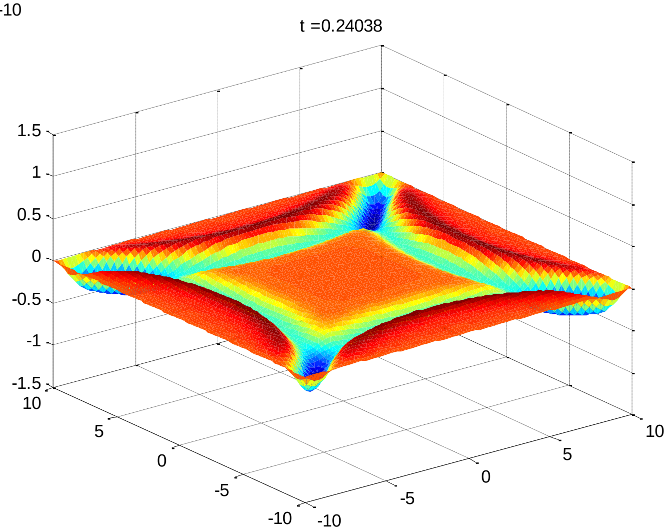
for $s = ({}^{c\,\Delta t}/_{\Delta x})^2$.

- These equations fully determine the mesh $U_{i,j}$.

Plots of the two Dimension Solution







Posing the Problem

- Again considering the wave equation for a density function $u(t;\ \vec{x})$ on some bounded domain Ω

$$\nabla^2 u = \ddot{u}/_{c^2}$$

- Now, ask the deceptively hard question:

  "Is it possible to trick the solution into behaving on the interior of Ω exactly like the domain extends without bound?"

- In one dimension,
"Yes, using absorbing boundary conditions (ABC)"
- In multiple dimensions,
"Simple extensions of ABCs seem not to suffice"

Perfectly Matched Layers (PML)

- In a 1994 Paper, Jean-Pierre Berenger outlines method for an absorbing layer, adjacent to the boundary, which applies an exponential decay to incident and departing waves

- There are many formulations of methods Berenger developed, but the process can be described as a coordinate transform

$$\boxed{\frac{\partial}{\partial x} \to \frac{1}{1 + i\frac{\sigma(x)}{\omega}} \frac{\partial}{\partial x}}$$

where $\sigma > 0$ in the abortion layer and $\sigma = 0$ otherwise.

Current Research in Cloaking

- Water ripples, sound, light, and electromagnetic radiation are all waves

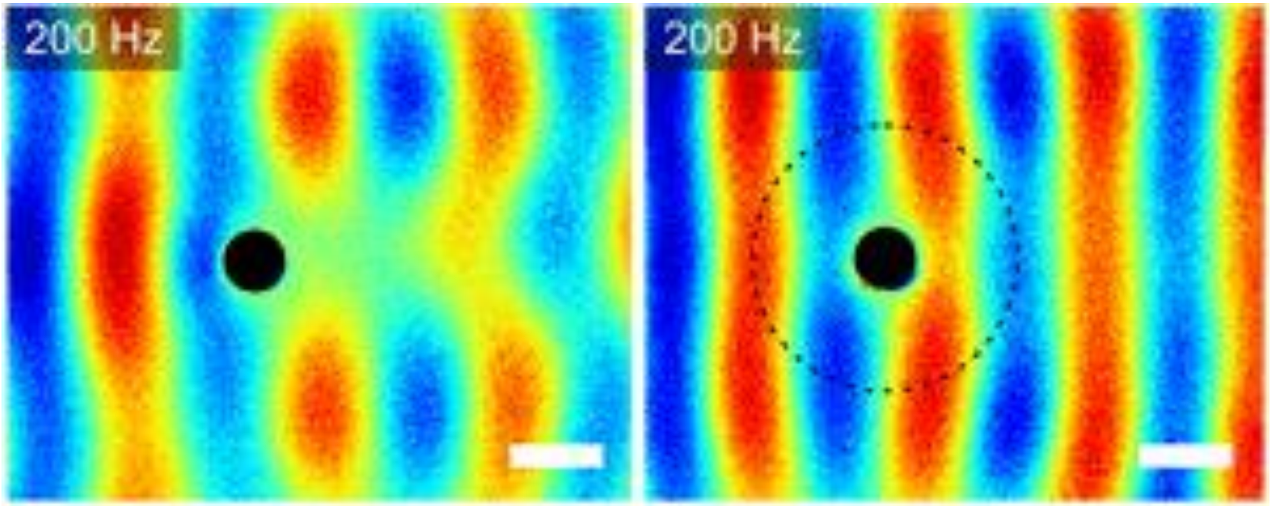


Image from N. Stenger, et al. (2012)

- The above images show traveling waves on an elastic surface containing a hole --with and without a local absorbing layer.

References

I. J.-P. Bérenger, “A perfectly matched layer for the absorption of electromagnetic waves,” J. Comput. Phys., vol. 114, no. 1, pp. 185–200, 1994.

II. N. Stenger, M. Wilhelm, and M. Wegener, “Experiments on Elastic Cloaking in Thin Plates,“ Phys. Rev. Lett. **108**, 014301 (2012), http://dx.doi.org/10.1103/PhysRevLett.108.014301