Competition Graphs

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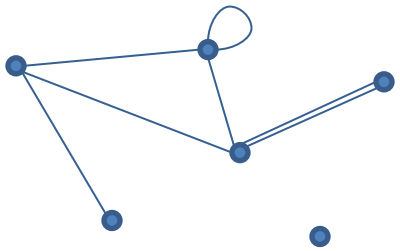
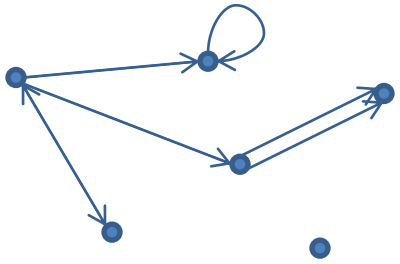
Graph Theory

- What is a graph?
  - A set of vertices and connecting edges
    - Loops, multiedges, isolated vertices
    - Vertices connected by an edge are *adjacent*



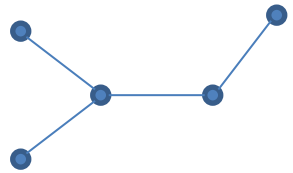
- Directed graph
  - Each edge has an associated direction

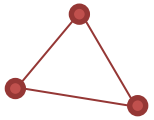


Properties of Graphs

- Subgraph
  - A graph $H = (S, A)$ is a subgraph of $G = (V, E)$ if $S \subseteq V$ and all adjacency relationships of G restricted to this subset are preserved in H

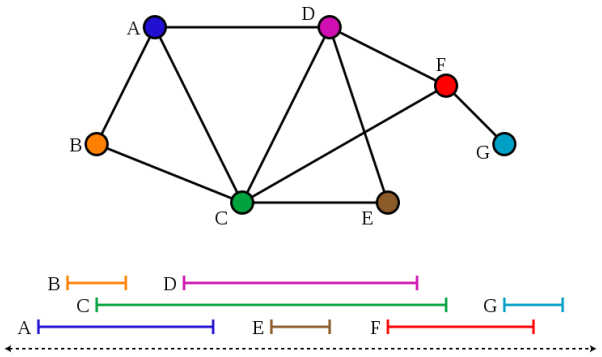
- Forbidden subgraph
  - A graph H is forbidden in G if it is not isomorphic to any subgraph of G
  - Forbidden subgraph characterization
    - Trees: connected graphs with no cycles





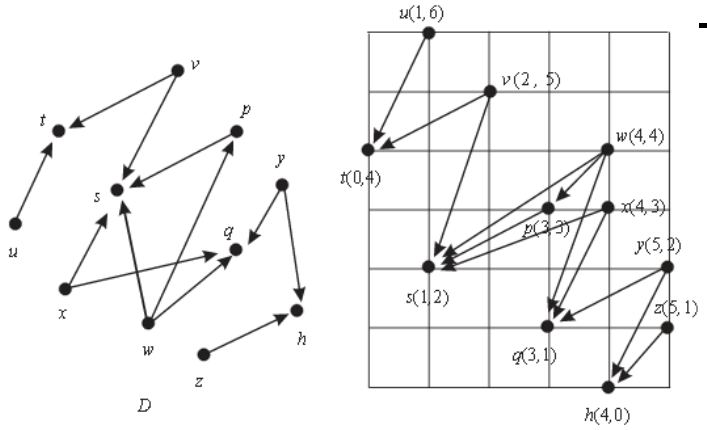
Geometric Graph Theory

- We can assign coordinate values to vertices
  - Interval graphs
    - Intersection graph of an interval on the real line
    - Known list of forbidden subgraphs (Lekkerkerker and Boland, 1962)



Example interval graph. Reproduced from Wikipedia.

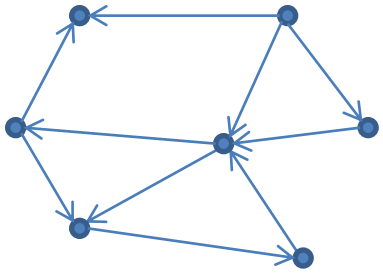
  - Partial orders
    - A set ordered according to a binary relation, which does not necessarily apply to all pairs in the set
    - Each vertex has a coordinate value



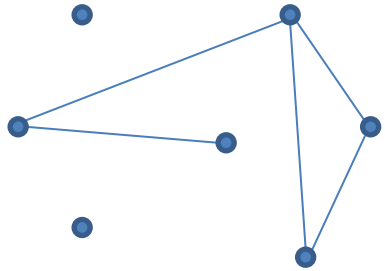
Example partial order graph. Reproduced from Cho & Kim (2005).

Competition Graphs

- The competition graph $C(D)$ of a digraph $D = (V,A)$ has the same vertex set as D, and two distinct vertices x, y are adjacent in $C(D)$ if there is some vertex $z \in V$ (possibly x or y) such that $xz, yz \in A$



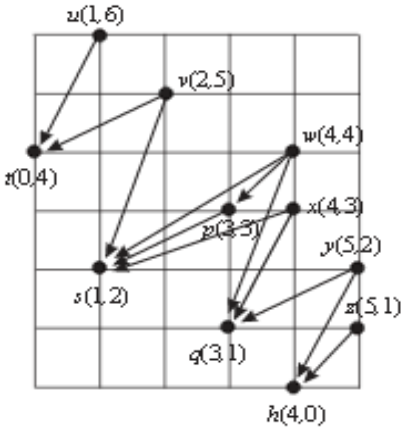
Digraph D

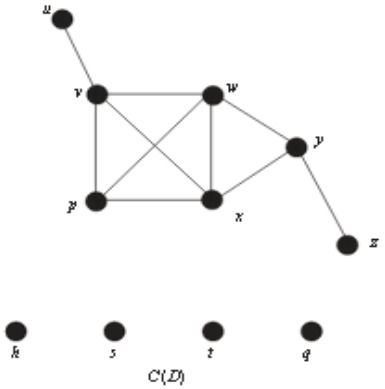


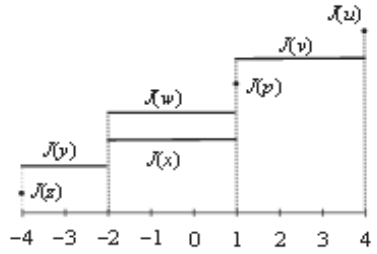
Competition graph $C(D)$

Properties of Competition Graphs

- Competition graphs of interval graphs are interval (Langley, Lundgren, & Merz, 1995)
- Competition graphs of a digraph of a doubly partial order are interval graphs (Cho & Kim, 2005)







A partial order graph, its competition graph, and the interval produced from the competition graph. Reproduced from Cho & Kim (2005).

The Problem

- Let V be a finite set of $\mathbf{R}^n$. Let D be a digraph such that $V(D) = V$ and there is an edge from $(a_1, a_2, \ldots, a_n)$ to $(b_1, b_2, \ldots, b_n)$ if and only if $a_i > b_i \; \forall i$.
  - Partial order graph

- Can you list forbidden subgraphs of the competition graph $C(D)$ of D?

Proposed Approach

- For $n = 2$, the competition graph is interval, so the list of forbidden subgraphs is known
- Investigate competition graphs for $n = 3$
  - Attempt to show these are also interval
  - Look for other characterizations of these graphs

References

Cho, H. H. & Kim, S. (2005). A class of acyclic digraphs with interval competition graphs. *Discrete Applied Mathematics*, *148*, 171-180.

Langley, L., Lundgren, J. R., & Merz, S. K. (1995). The competition graphs of interval digraphs. *Congr. Numer*, *107*, 37-40.

Lekkerkerker, C. G. & Boland, J. C. (1962). Representation of a finite graph by a set of intervals on the real line. *Fund. Math. 51*, 45-64.