Forbidden Subgraphs of Competition Graphs

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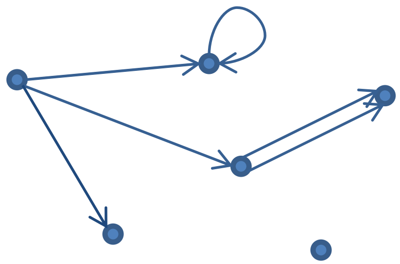
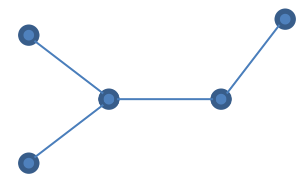
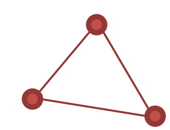
Graph Theory

- Directed graph
  - Each edge has an associated direction
  - Every directed, acyclic graph has at least one source and one sink node



- Subgraph
  - A graph $H = (S, A)$ is a subgraph of $G = (V, E)$ if $S \subseteq V$ and all adjacency relationships of G restricted to this subset are preserved in H
- Forbidden subgraph
  - A graph H is forbidden in G if it is not isomorphic to any subgraph of G
  - Forbidden subgraph characterization
    - Trees: connected graphs with no cycles





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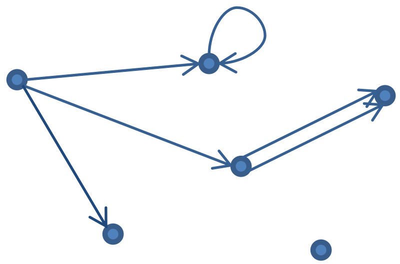

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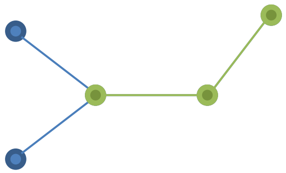
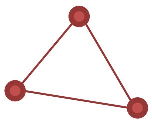
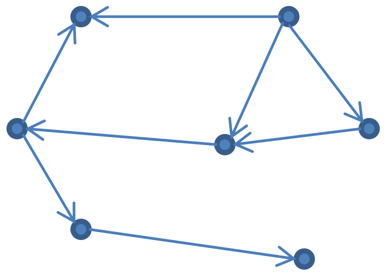


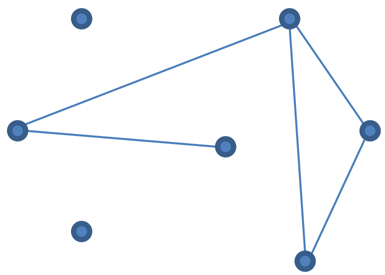


Competition Graphs

- The competition graph $C(D)$ of a digraph $D = (V,A)$ has the same vertex set as D, and two distinct vertices x, y are adjacent in $C(D)$ if there is some vertex $z \in V$ (possibly x or y) such that $xz, yz \in A$



Digraph D

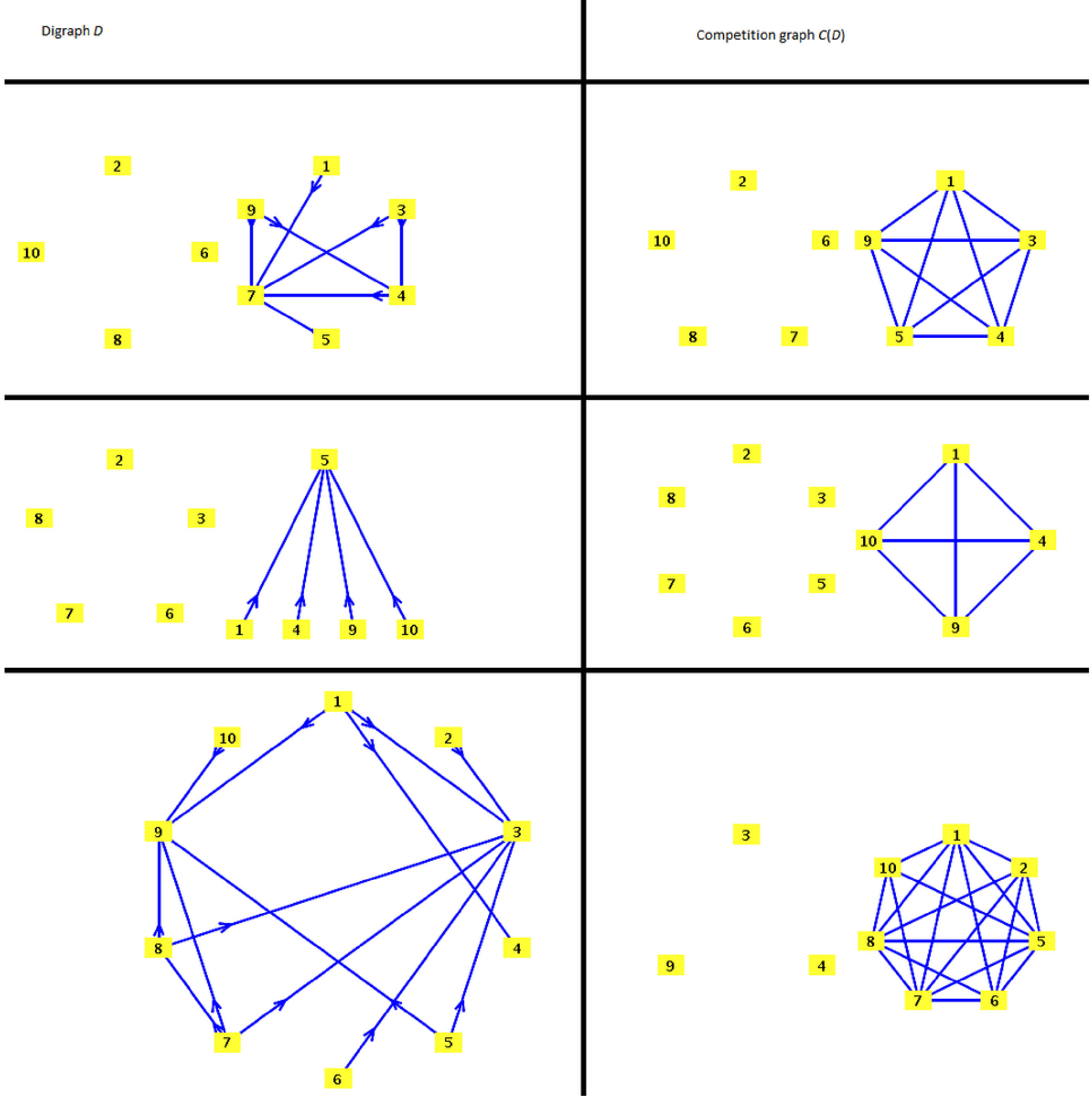


Competition graph $C(D)$

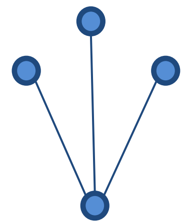
The Problem

- Let V be a finite set of $\mathbf{R}^n$. Let D be a digraph such that $V(D) = V$ and there is an edge from $(a_1, a_2, \ldots, a_n)$ to $(b_1, b_2, \ldots, b_n)$ if and only if $a_i > b_i \ \forall i$.

- Can you list forbidden subgraphs of the competition graph $C(D)$ of D?


Digraph D
Competition graph C(D)

Claw Graphs



- Claw Graph
  - A tree on nodes with one node having vertex degree n-1 and the other n-1 nodes having vertex degree 1
- Theorem: Any subgraph with maximum degree n-1 is forbidden in $C(D)$
  - Lemma: There is always one isolated vertex in $C(D)$
  - Let v be a vertex of degree n-1 in $C(D)$
  - There are no loops or multiedges, so there must be an edge between v and the other n-1 vertices, leaving no isolated vertices

Other Forbidden Subgraphs

- There are no forbidden subgraphs of $C(D)$ with maximum degree less than or equal to n-2
  - Construction of an example

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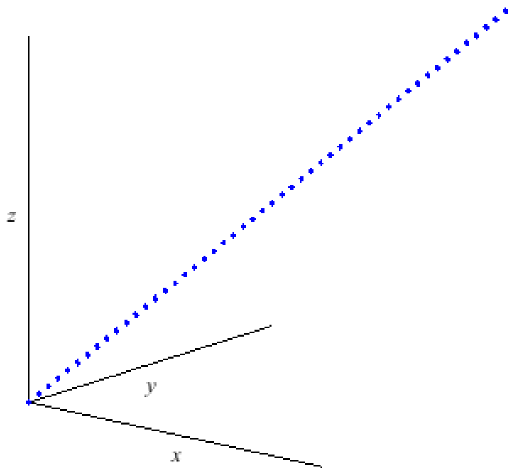

The line $x = y = z$ in $\mathbf{R}^3$

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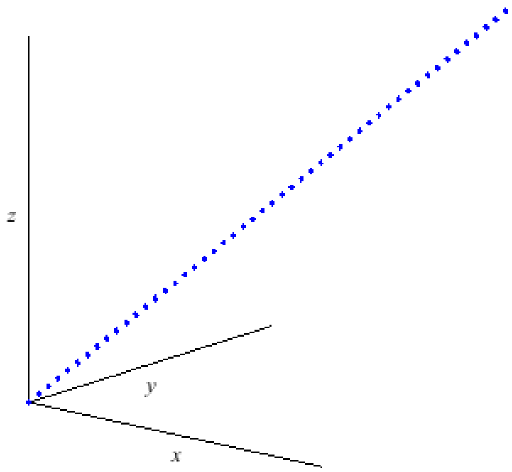

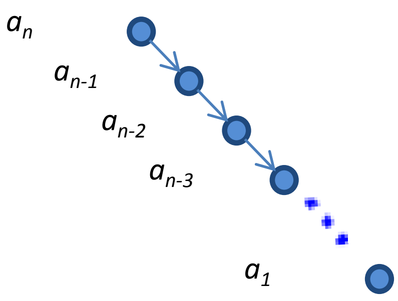

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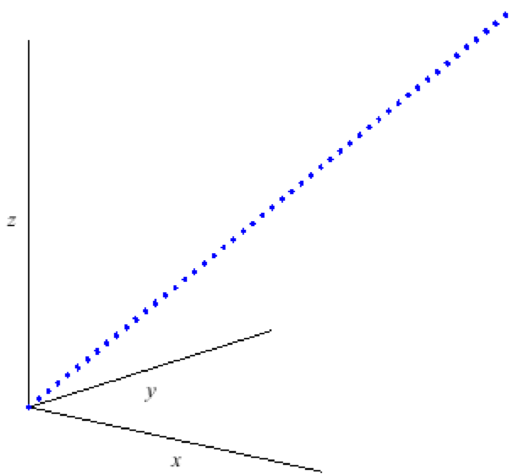

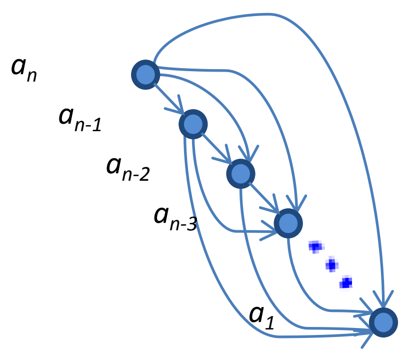

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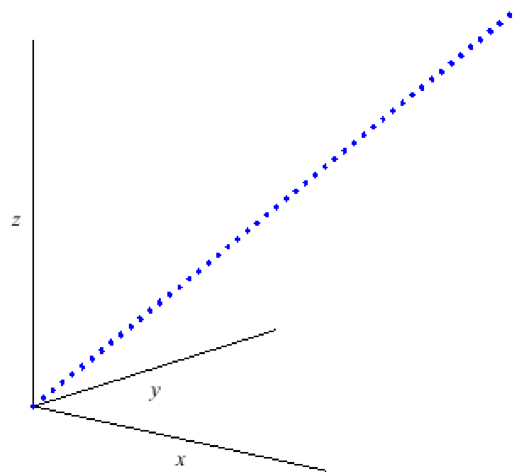

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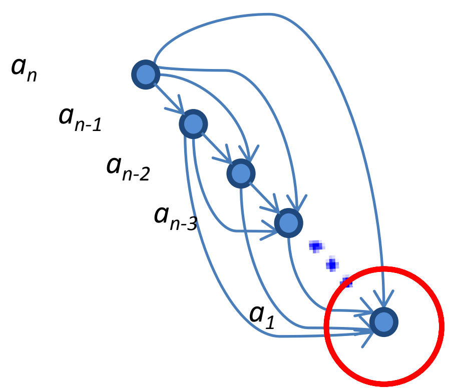

Digraph D – Note that a_1 is receiving arcs from all other vertices

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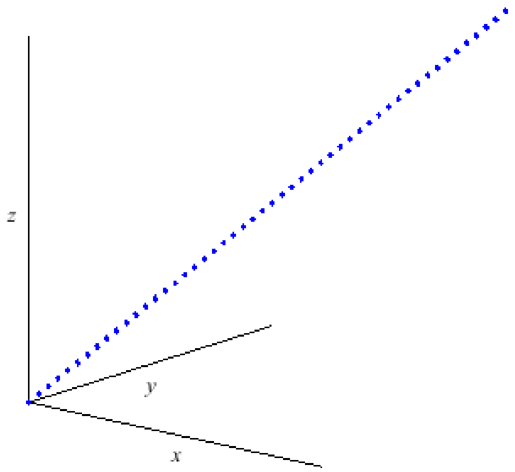

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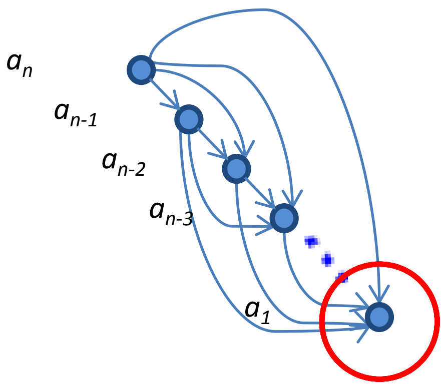

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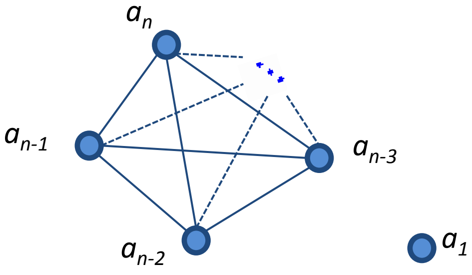

Competition graph $C(D)$ is a complete K_{n-2} with a_1 isolated

Domain Restriction

- Suppose you have two lines through the origin $y = ax$, $y = bx$, where $a \neq b$, and suppose you distribute n points on these two lines.
- What are the forbidden subgraphs of the competition graphs generated from our relation?

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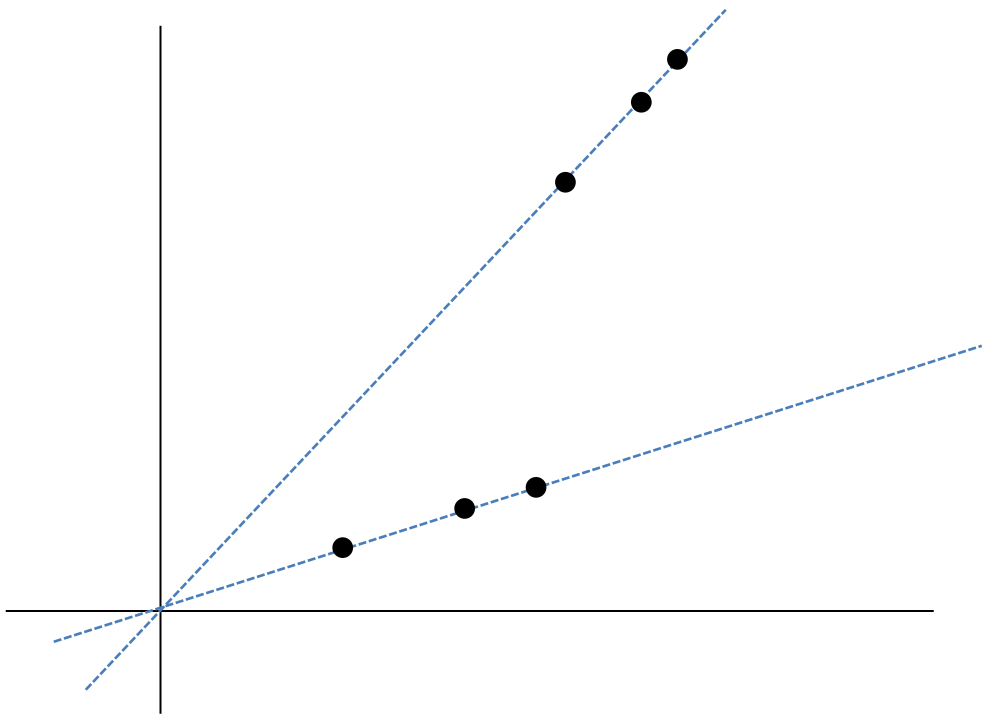

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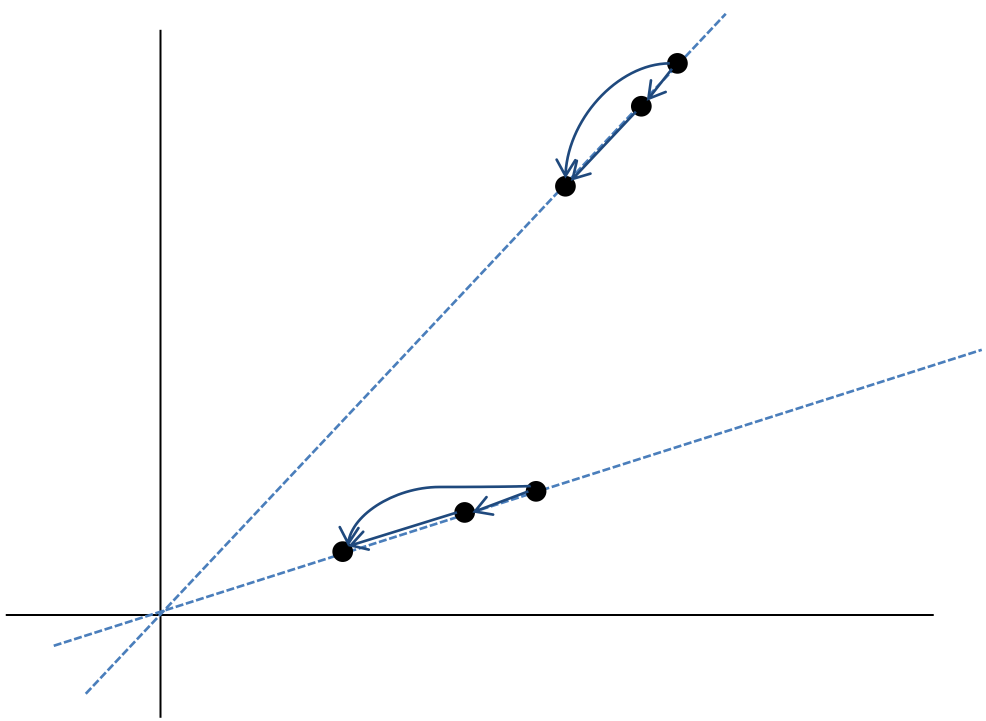

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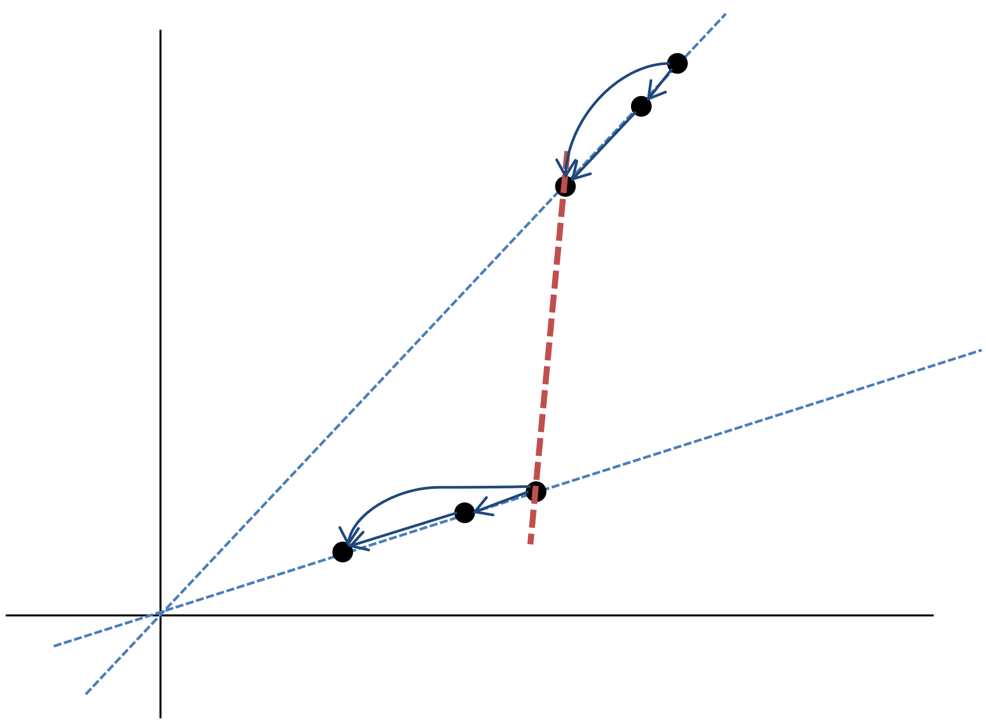

Dominating line - A line of positive slope connecting points on two different lines

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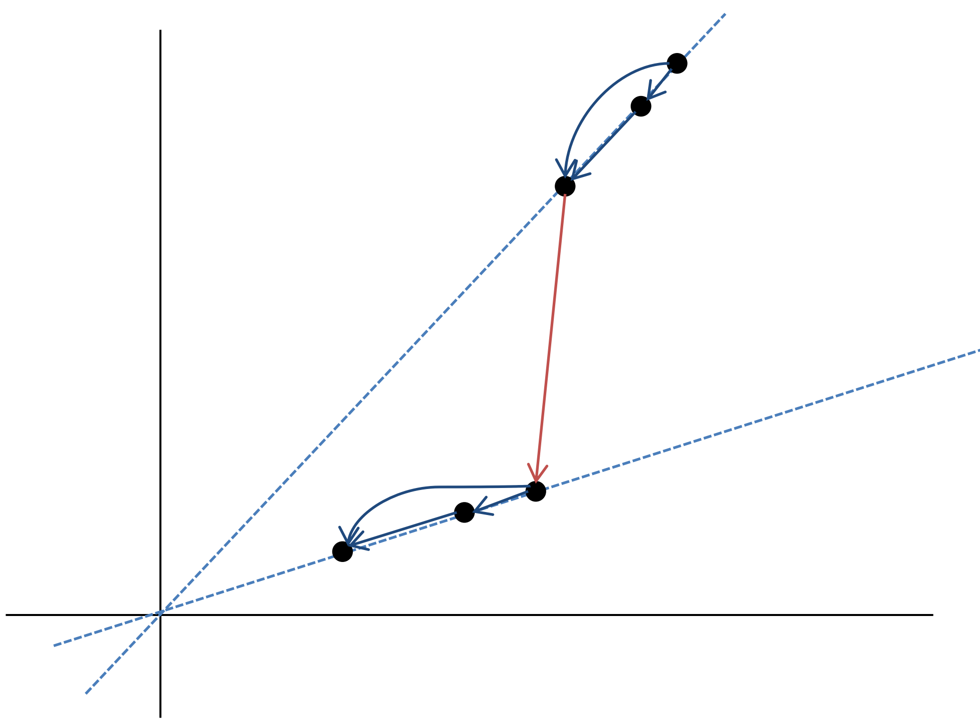

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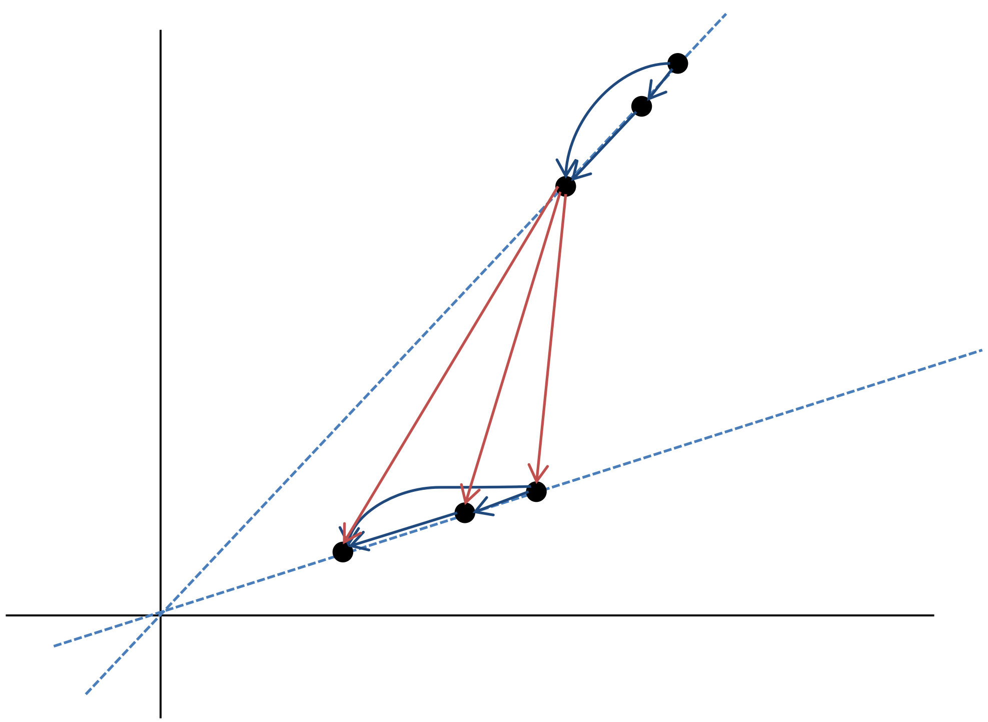

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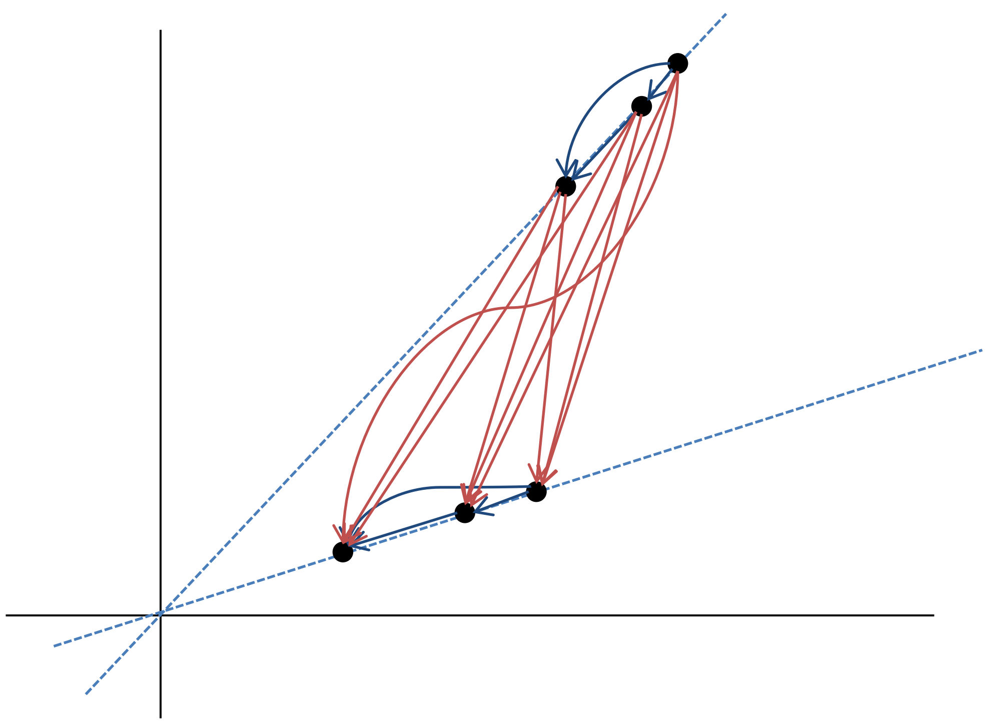

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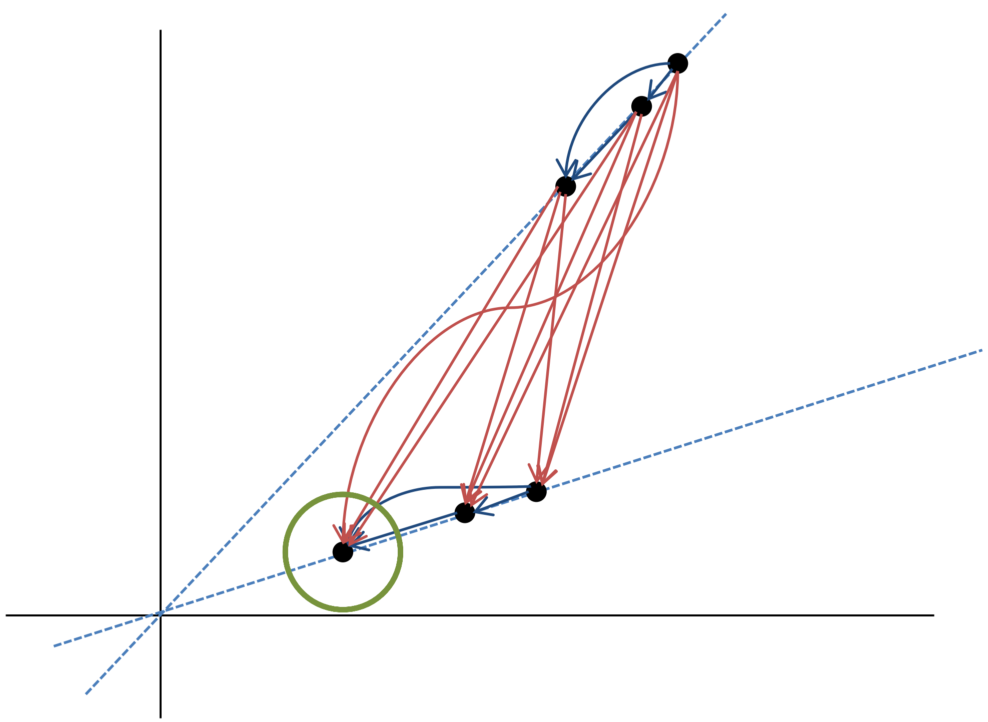

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If there is a dominating line between the point with the largest x value on one line and the point with the smallest x value on the other, the competition graph is a complete K_{n-2}

Future Work

- Other cases in the domain restriction
- Domain restriction is currently only in two dimensions
  - Extend it to three dimensions, *k* dimensions
- Increase the number of lines
- Consider other domain restrictions
  - Points distributed in a small grid in $\mathbf{R}^2$