Data Visualization Using Polynomiography

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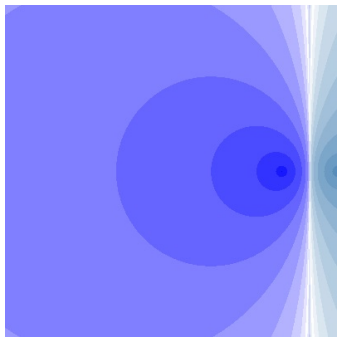
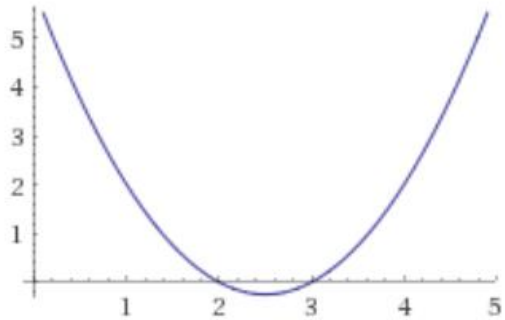
Introduction to Polynomiography

What is polynomiography?

- "The art and science of visualization in approximation of the zeros of complex polynomials, via fractal and non-fractal images created using the mathematical convergence properties of iteration functions" (Kalantari)
- The polynomiography software is a tool, like a camera, and can be used to capture beautiful images of the a new way of looking at complex polynomials.



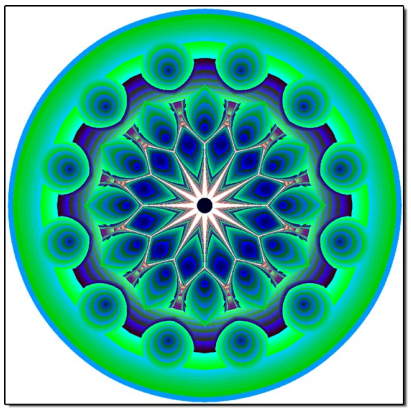


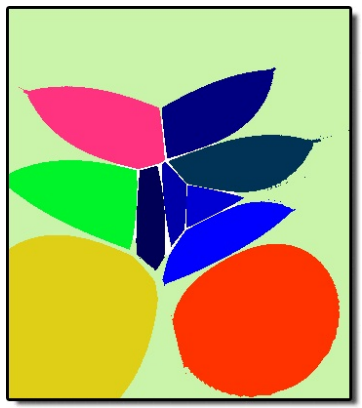
images of $z^2 - 5z + 6$ in the polynomiography software vs. plotted on a standard graph

Mathematics Behind Polynomiography

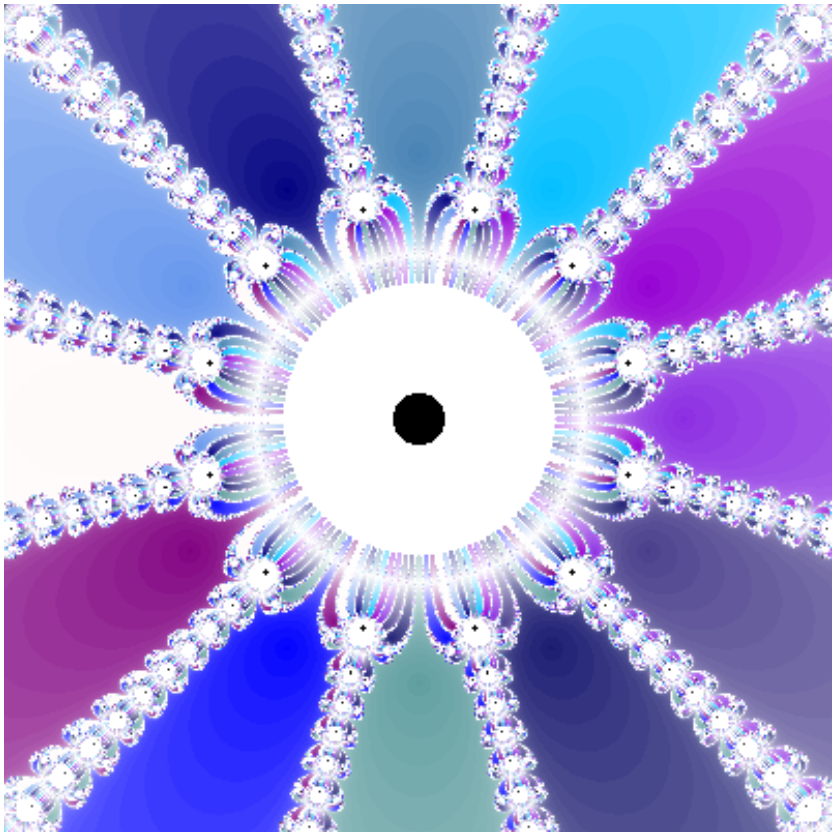
- Based on iteration functions designed to approximate the roots of polynomials (takes an input, creates an output which becomes the next input until convergence is achieved)
- Best known: Newton's Method:

$$N(z) = z - \frac{p(z)}{p'(z)}$$





Example: Zoomed in polynomiograph of $z^{12} - 1$



Polynomiography as a Visualization of Data

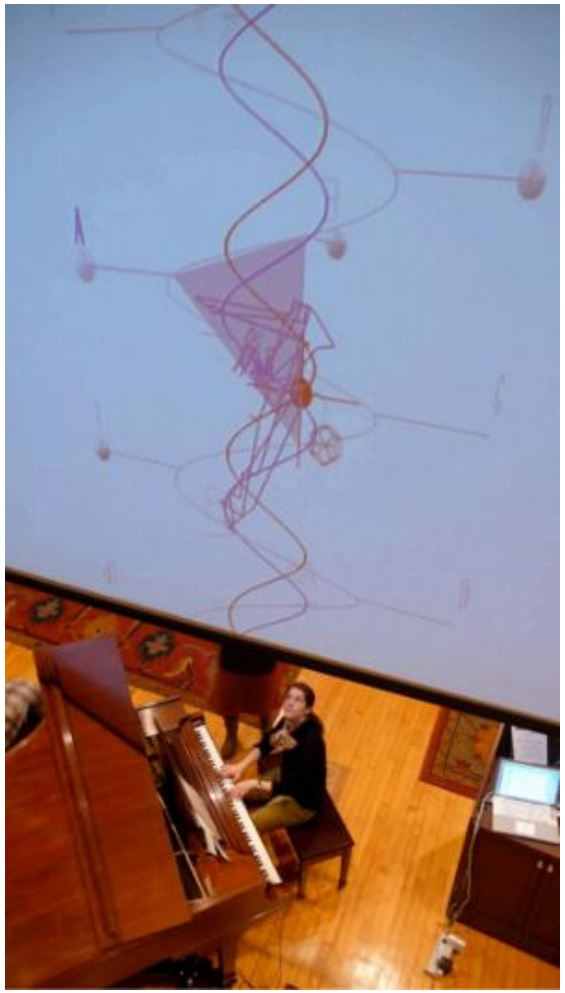
In the current polynomiography software, there is:

- Input as a polynomial equation (default)
- Input as roots to a polynomial (creates a polynomial based on the roots)
- Input as a number (converts number to a polynomial - each digit is mapped to a coefficient)
- Input as a word (converts word to a polynomial)
- Potential research: What about input as a melody? Or composition? What information about a piece of music can be stored mathematically? How can the data be compressed while maintaining the character of the piece? How can this enhance the listening experience?

Current Music Visualization Tools

MuSA.RT: Music on the Spiral Array, Real Time

- Stores pitch classes in a spiral array
- Real time interpretation of MIDI input
- Key finding algorithm - nearest neighbor search based on the MIDI input
- Displays the visual information and tracks the tonal structures as they appear



Current Music Visualization Tools

Narratives 2.0

- Draws the still image in real time
- Analyzes the frequencies of the music
- Does not present inherent information about the piece

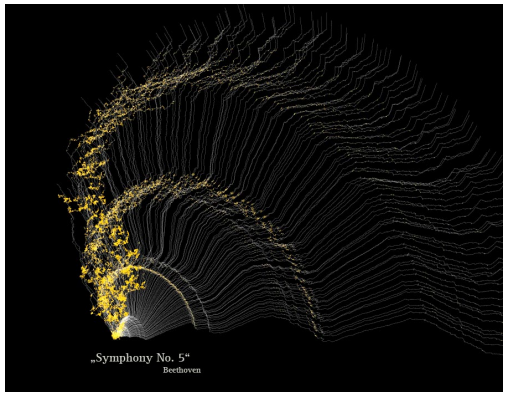


image for Beethoven's Fifth Symphony

The Shape of Song

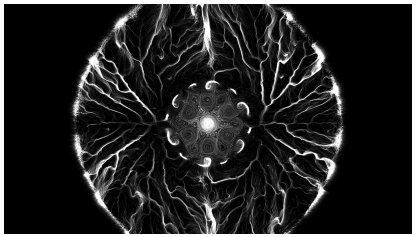
- Analyzes MIDI files and searches for repeated patterns (repeated substrings in the melody)
- Visualization of the form: displays arcs connecting the repeated sections

image for Beethoven's Fifth Symphony

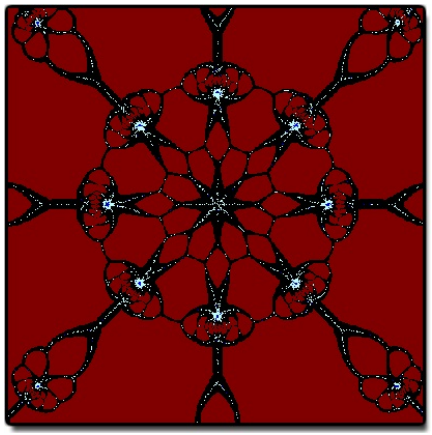
Current Music Visualization Tools

Abstract Birds: Genesi

- Animation created in real time during a live audio/visual show
- Software extracts data from electronic instruments to create an environment that changes in response to the music, like a videogame
- Generative music algorithms: always changing, unrepeatable images, depends on the artist's choices



still image from *Genesi*, created by Abstract Birds with their own software



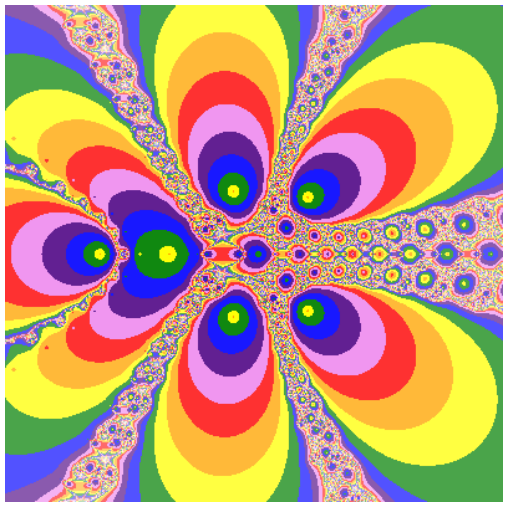
Acrobat, created using polynomiography by Dr. Kalantari

Why Polynomiography?

- Art: The software creates beautiful images and animations (showing shifts from one polynomial to another). Music depends upon the progression of sounds over time.
- Standard Encoding: A feature already exists to map words to polynomials. Goal: a similar mathematical mapping from sound to an image while maintaining the characteristics of the piece - key, instrumentation, etc.
- Polynomiographs offer more control than fractal images this way of viewing music would offer more information about the piece.

An Elementary Image Example

- "Happy Birthday" in the key of C created with making the frequency of notes the coefficients in a polynomial.
- Frequencies: C:5/25, G:9/25, E:2/25, F:2/25, A:3/25, D:2/25, B:2/25
- Polynomial: $5/25z^6+9/25z^5+2/25z^4+2/25z^3+3/25z^2+2/25z+2/25$
- Logic: higher degree of z corresponds to a more consonant interval



Possible Plans for Implementation

Method 1: Manually make the animation or picture based on a score and a mapping system.

- Major work to be done: creating the mapping system to incorporate several inherent aspects of the piece (ie: key, instrumentation, shifts of tonality).
- Possible mapping for instrumentation: each instrument receives their own factor. The more instruments, the more rings in the polynomiograph, capturing the complexity of the orchestration.

Possible Plans for Implementation

Method 2: Take information from a midi file and produce a polynomiograph (or animation).

- Major work to be done: Both retrieving data from the midi file and mapping it to the output of the polynomiograph software. This requires both a mapping system (as above) and an automated program that can access the current polynomiography software.
- Challenges: Some information notated in the score and retrievable through music theory analysis is not contained in the midi file (ex: surprisingly, the key.) Dr. Elaine Chew, formerly of USC, worked on the key finding program for MuSA.RT that is available as an app, so that would be worth investigating.

Possible Applications to Music Cognition

- Enhancing the listeners experience with visual cues
- Creating a new art form that uses mathematics to translate from sound to images
- Enabling those with hearing handicaps to experience music in some form
- Assisting with patients who have cochlear implants
- Other applications of data visualization using polynomiography

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