Visualization of the Triangle Algorithm in R2

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DIMACS Student Presentations

The Elephant in the Room

Minor Setback: My laptop sadly died last Saturday, taking all of my data with it (corrupt hard drive).

This week was spent completely re-coding from scratch and developing the images I'd been working on. As such, there is more to do (next week, and hopefully for an article).

What is the Triangle Algorithm?

The problem: Given a set of points $S = \{v_1 ... v_n\}$ in R^m and a point p in R^m, is p in the convex hull of S?

What is the Triangle Algorithm?

Dr. Kalantari's Triangle Algorithm:

- Given $p' \in S \backslash \{p\}$, check if there is a pivot point $v_j \in S$ (if $d(p', v_j) \geq d(p, v_j)$). If not, then p' is a witness. Stop.
- Otherwise, continue, and move to the point closest to p along the line between p' and v_j. My code finds the v_j which will produce the new point closest to p out of all the possible v_j.

What is the Triangle Algorithm?

- The step size: $\alpha = \frac{(p-p')^T(v_j-p')}{d^2(v_j,p')}$
- New $p' = \begin{cases} (1-\alpha)p' + \alpha v_j & \text{if } \alpha \in [0,1] \\ v_j & \text{otherwise} \end{cases}$
- Iterate until the point is inside an epsilon tolerance or a witness is found

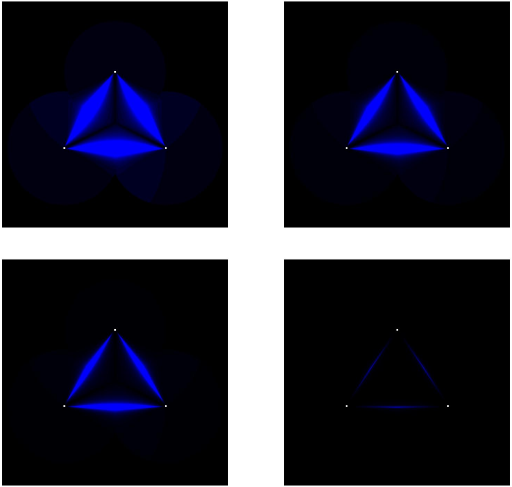
Visualization of the Algorithm

- Coding the algorithm in R^2
- Starting with simple polygons for the convex hull - input points
- Scales the convex hull area to fit in the image area - some points will be inside, some will not
- Each point in the image area is given as p, and the Triangle Algorithm is run on that point to determine whether or not it is inside the convex hull
- p' always starts at the center of the convex hull (for now)

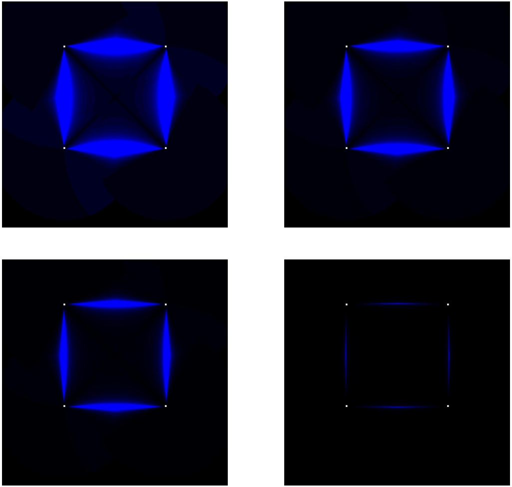
Viewing Iterations

- The code counts the number of iterations needed to determine whether the point p is inside the convex hull (inside the epsilon tolerance) or outside the convex hull (where p prime is a witness)
- Many of the points can be determined very quickly. However, along the edges (particularly the midpoints), the algorithm takes many more iterations.
- The challenge for visualization: Showing the layers of initial iterations while preserving the few points that take a significantly greater number of iterations.

Iteration Evolution: Triangle Gradient

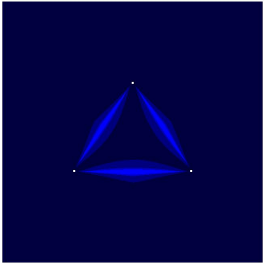


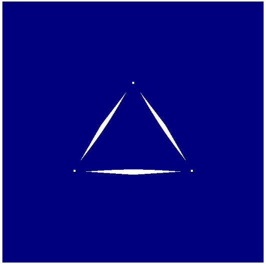
Iteration Evolution: Square Gradient



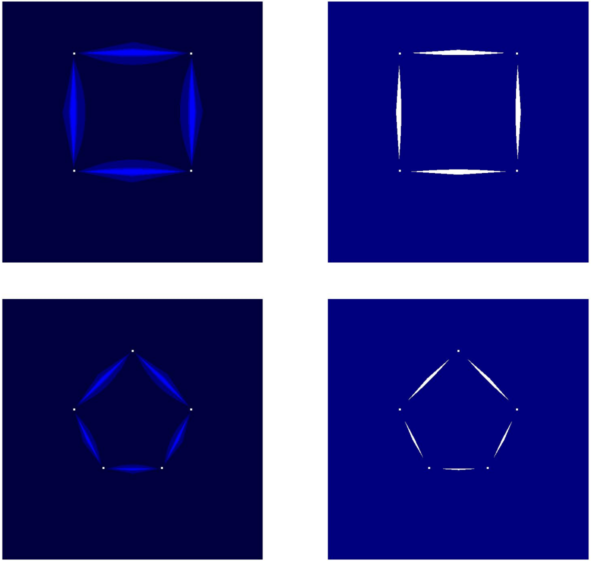
Viewing Bands of Iterations

Left: 0-10, 10-100, 100+
Right: 0-100, 100+

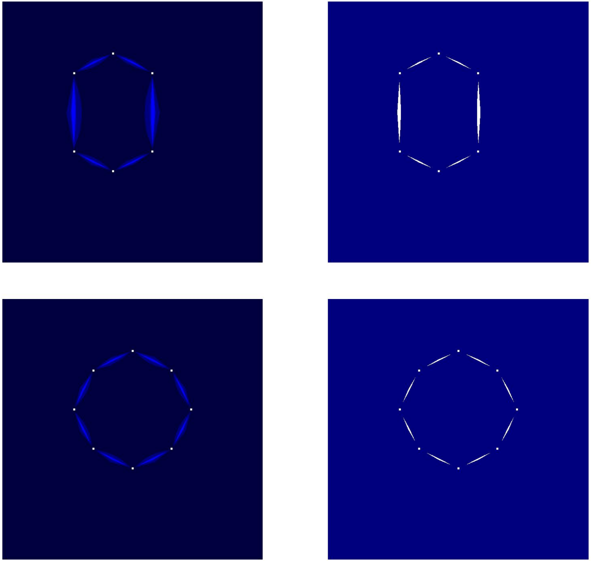




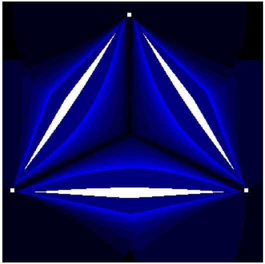
Viewing Bands of Iterations

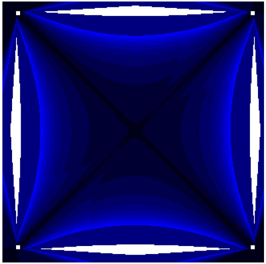


Viewing Bands of Iterations



Zooming in on Initial Iterates (Less than 10)

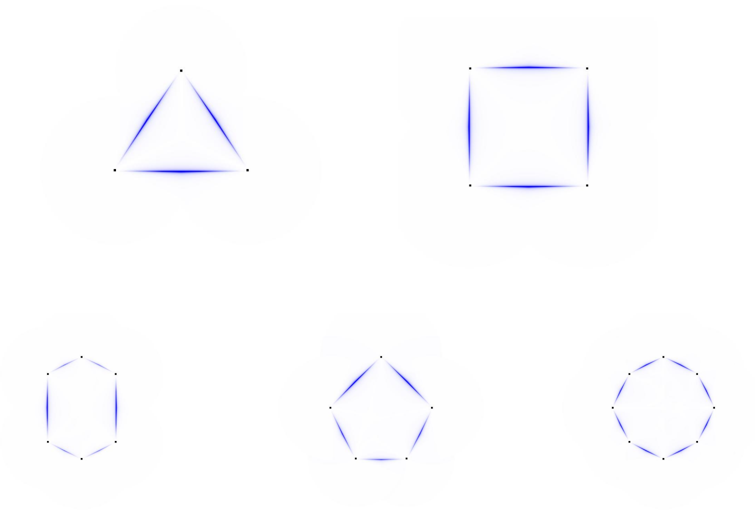




Applications: Education

- Introduces students to the idea of a convex hull
- Seeing is believing, easier to understand
- Good for younger students to encourage mathematics education

Showing the Convex Hull



Applications: Algorithmic Artwork

- Inspired by Dr. Kalantari's work in polynomiography
- Triangle Algorithm is another iteration function
- Can create interesting images based on coloring iterations or bands of iteration
- Currently working with simple polygons
- Trend: Symmetry!
- Future work: any set which forms a convex hull

Artwork

Series 1: Sunshine (1)

Artwork

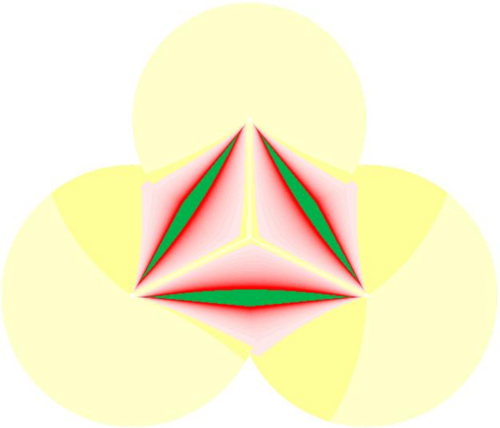
Series 1: Sunshine (2)

Artwork

Series 1: Sunshine (3)

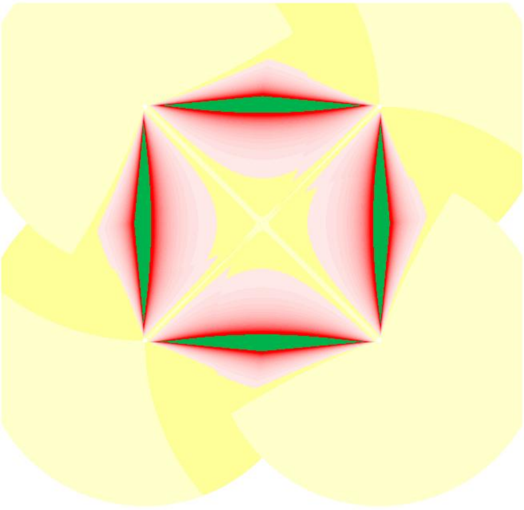
Artwork

Series 2: Rose (1)



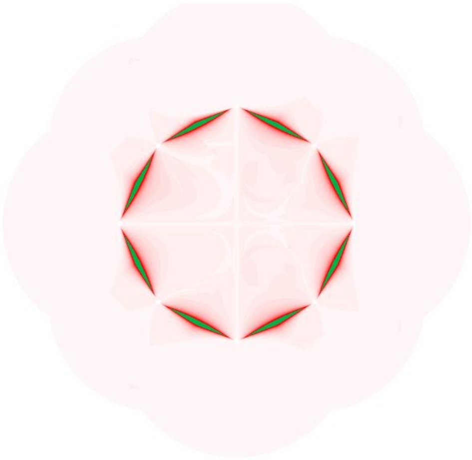
Artwork

Series 2: Rose (2)



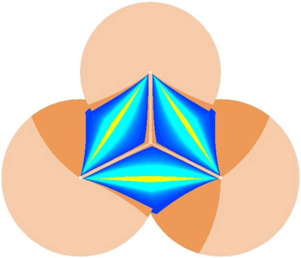
Artwork

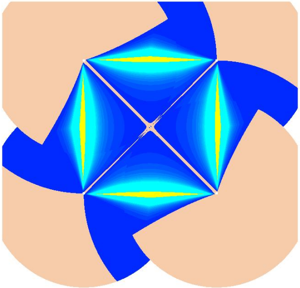
Series 2: Rose (3)



Artwork

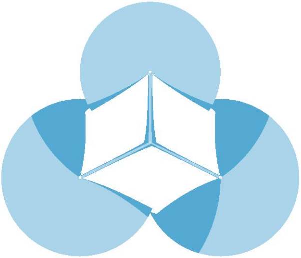
Series 3: Beach (1)





Artwork

Series 4: Corners of the Sky



Future Research

- Improving the pivot point selection
- Improving the midpoint weakness (adding in more possible pivots to the set, for example midpoints of all edges)
- Developing images for different epsilon tolerances (all of these were created with a 10^{-4} tolerance)
- Dual: Here, p prime was kept as the center of the convex hull while p changed. Next: start working on p prime changing while p stays the center. Also: different initial starting points for both.
- More than one p? Choose from two randomly.
- More complicated convex hull shapes
- More artwork!

The End

Thank you for your attention! Any questions?