Peeling in graphs

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Definition of peeling

- Peeling number of the vertex *v* is the number *peel(v)* defined by the following process.
  1. p = 0
  2. Repeat
     1. While exists vertex v such that *deg(v)* <= p
        1. *peel(v)* = p
        2. remove v
     2. p = p+1
     3. End if no vertex remains

Properties

- Peeling numbers are unambiguous.
- Vertices of graph are divided into layers.
- These layers are sometimes called *k-cores*
- *peel(v)* <= *deg(v)* …for every vertex *v*
- The maximal peeling number of a graph equals the degeneration of a graph.

Motivation of peeling

- Peel numbers are studied in large networks.
  - Internet network
  - Social network
- We want to determine the peel number of a given vertex online in a gradually evolving graph.
  - Our aim is to develop a data structure for this.

Data structure

- For a graph $G = (V, E)$ we want to regularly process the following queries.
  a) Initialize the structure by the graph G
  b) Add an edge (u,v) into G
  c) Delete an edge (u,v) from G
  d) Determine a peel number of a vertex v at the time t
- We want to achieve the best amortized time complexity for queries b), c), d) and the best time complexity for a one-time query a).

Characterization

- We want to characterize graphs which vertices all have the same peeling number. Lie in the same layer.
- We determine this class of graphs as GF_k
  - GF_1 is equivalent to trees
  - GF_2 is equivalent to ???
  - …
  - GF_k is equivalent to ???

References

- [1] Alberto Montresor, Francesco De Pellegrini, and Daniele Miorandi. Distributed k-core decomposition. CoRR, abs/1103.5320, 2011
- [2] Michael Baur, Marco Gaertler, Robert Görke, and Marcus Krug. Generating graphs with predefined k-core structure. Technical report, DELIS – Dynamically Evolving, Large-Scale Information Systems, 2007