Graph theory and Brain connectivity

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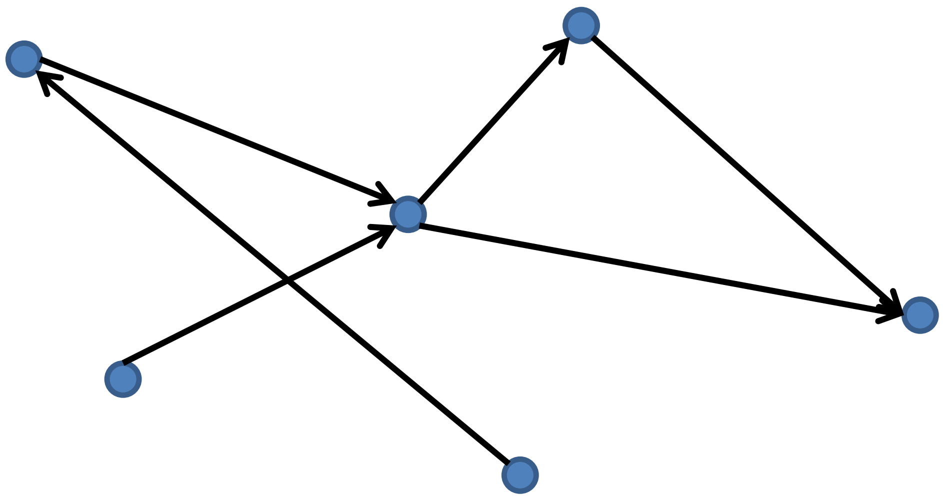
Graph Theory

A graph is a non empty set of vertices(or nodes) V and a set of edges E that connect pairs of nodes

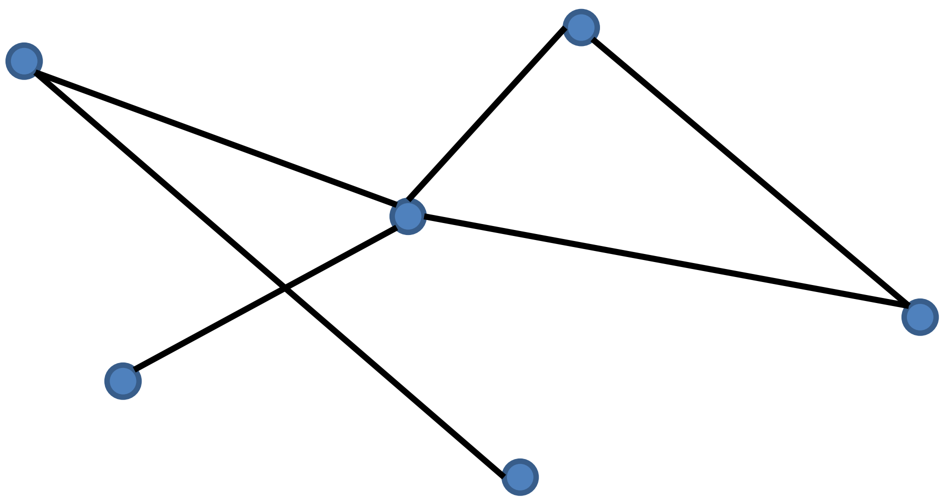
Directed Graph

- A directed graph is a graph where the vertices are joined by directed edges.



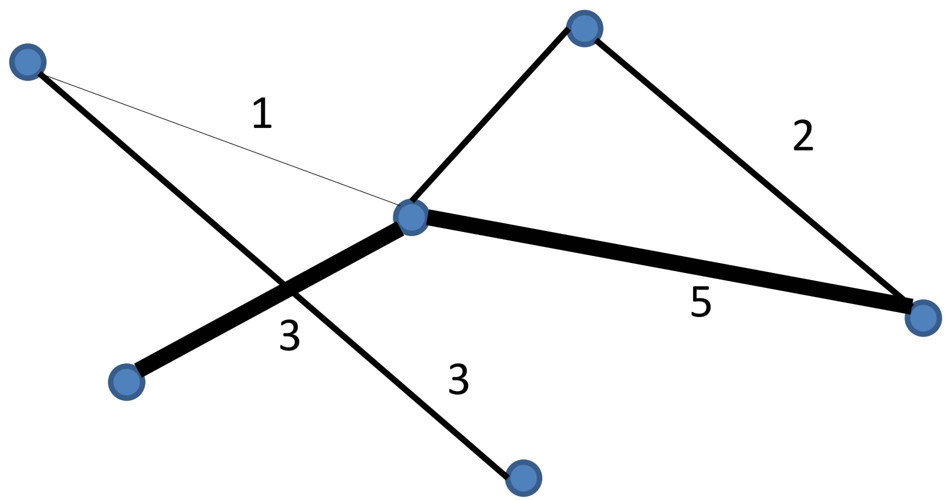
Undirected Graph

- An undirected graph is a graph where the nodes are connected with undirected edges.



Weighted Undirected Graph

- A graph can be weighted or unweighted. A weighted graph is a graph where the edges have weights assigned to them.



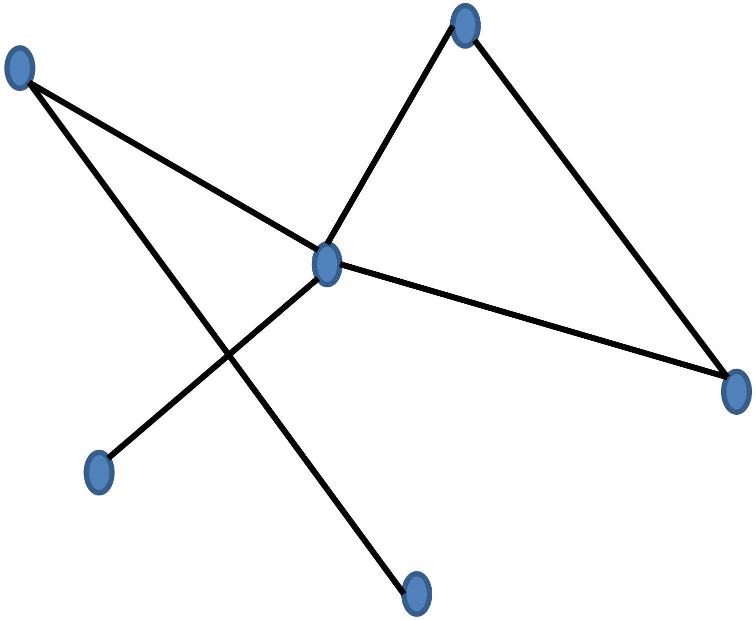
Degree

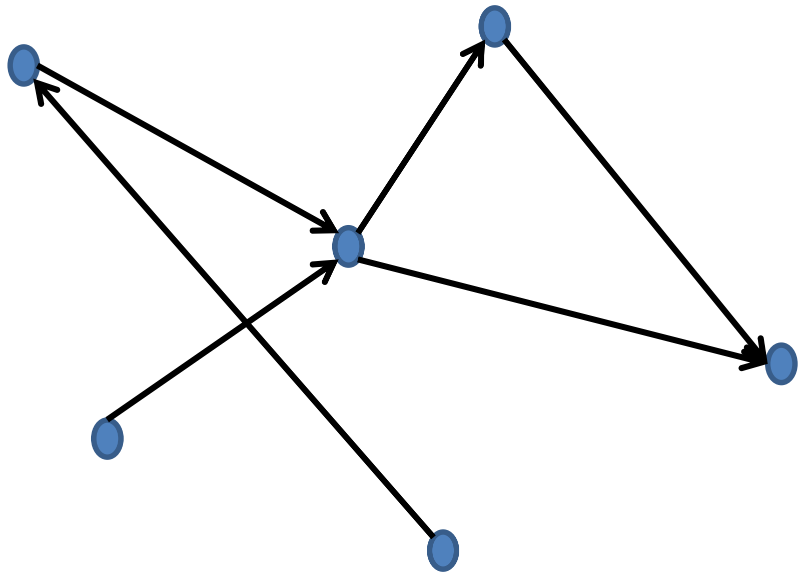
- Undirected graph: the number of edges connected to a node
- Directed graph: The *in-degree of a vertex v*,

denoted *deg*–(*v*), is the number of edges

directed into *v*.

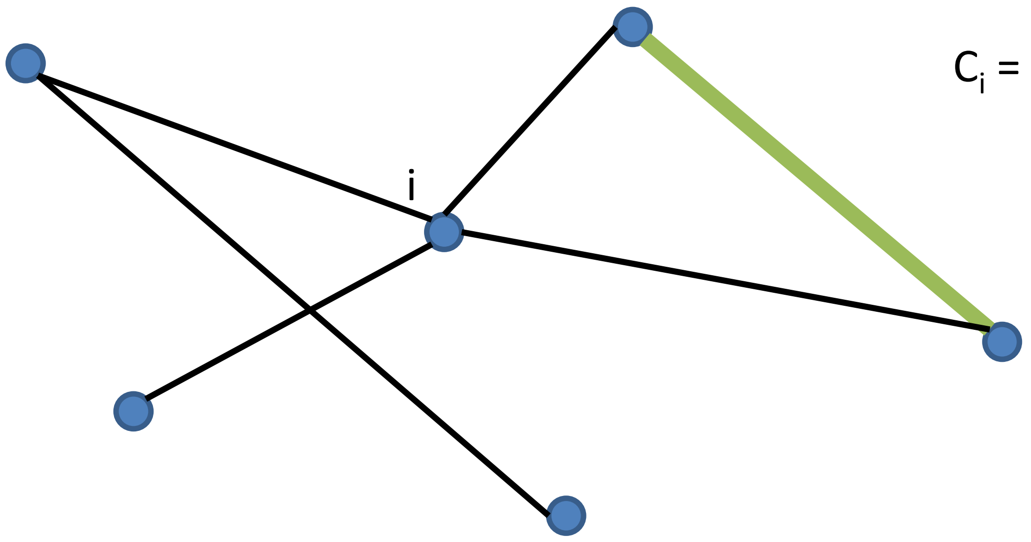
- The *out-degree of v*, denoted *deg+*(*v*)*,* is the number of edges directed out of v.





Clustering coefficient

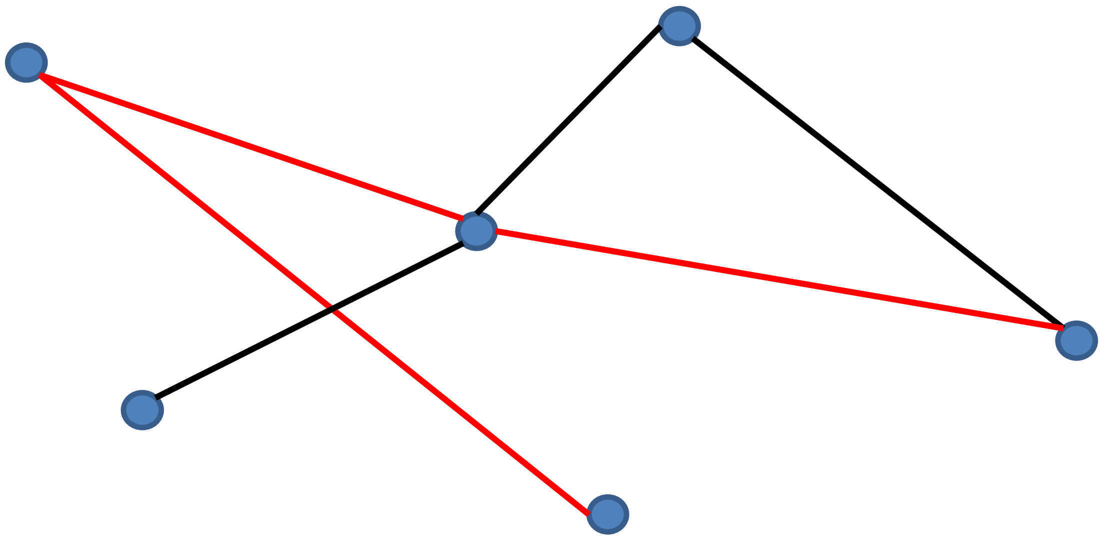
- $C_i = \dfrac{\text{\# of connections between neighbors of i}}{\text{\# of potential connections}}$



$C_i = 1/6$

Path length

- Unweighted: the number of edges between two nodes
  - Weighted: weights of edges must be taken into consideration
- Often we want to find the shortest path

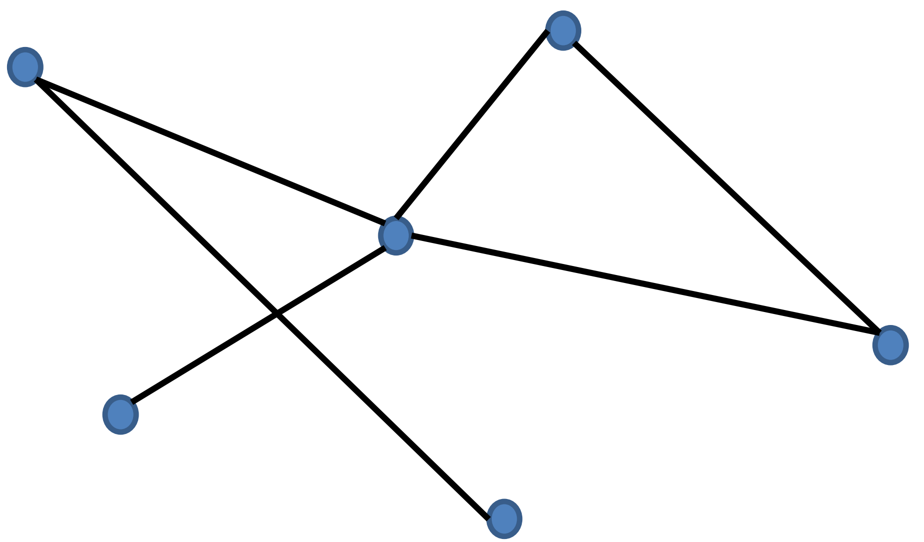


Betweenness

- Consider all pairs of distinct nodes in a graph and determine the shortest path length between them. The betweenness of a node is the frequency with which it occurs in all shortest paths.

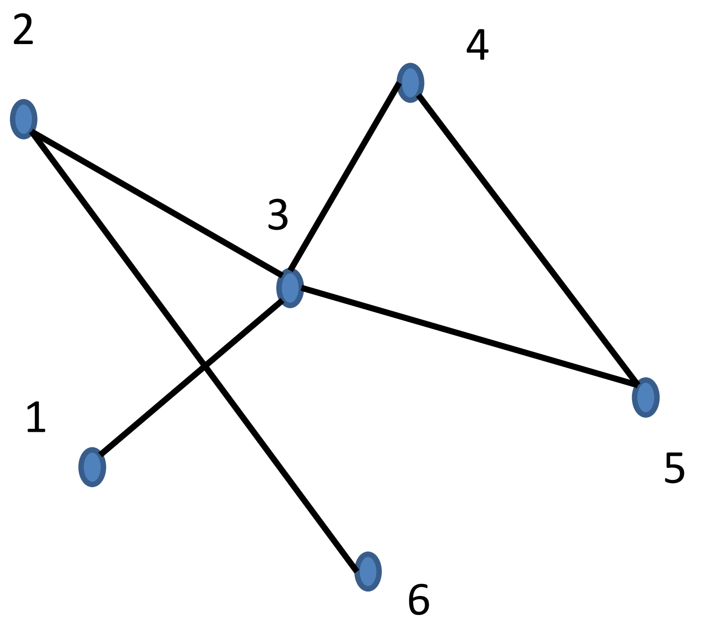
Edge density

- $E = \dfrac{\text{\# of existing edges}}{\text{\# of potential connections}}$



E = 6/15 = 0.4

Adjacency matrix



$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

- Adjacency matrix→Laplacian→Normalized Laplacian→→eigenvalues, eigenvectors for spectral analysis

Application of Graph

- We can say that almost “EVERYTHING IS A GRAPH”
- Graph theory is used today in the modelling of:
- Social networks
- Communications networks
- Information networks
- Software design
- Transportation networks
- Biological networks
- Etc.

Viewing brain connectivity as a graph

	Functional connectivity	Structural connectivity	Effective connectivity
Nodes	Functional regions of interest (ROIs)	Neurons	ROIs
Edges	Fiber tracts	Axons	Causal relationship

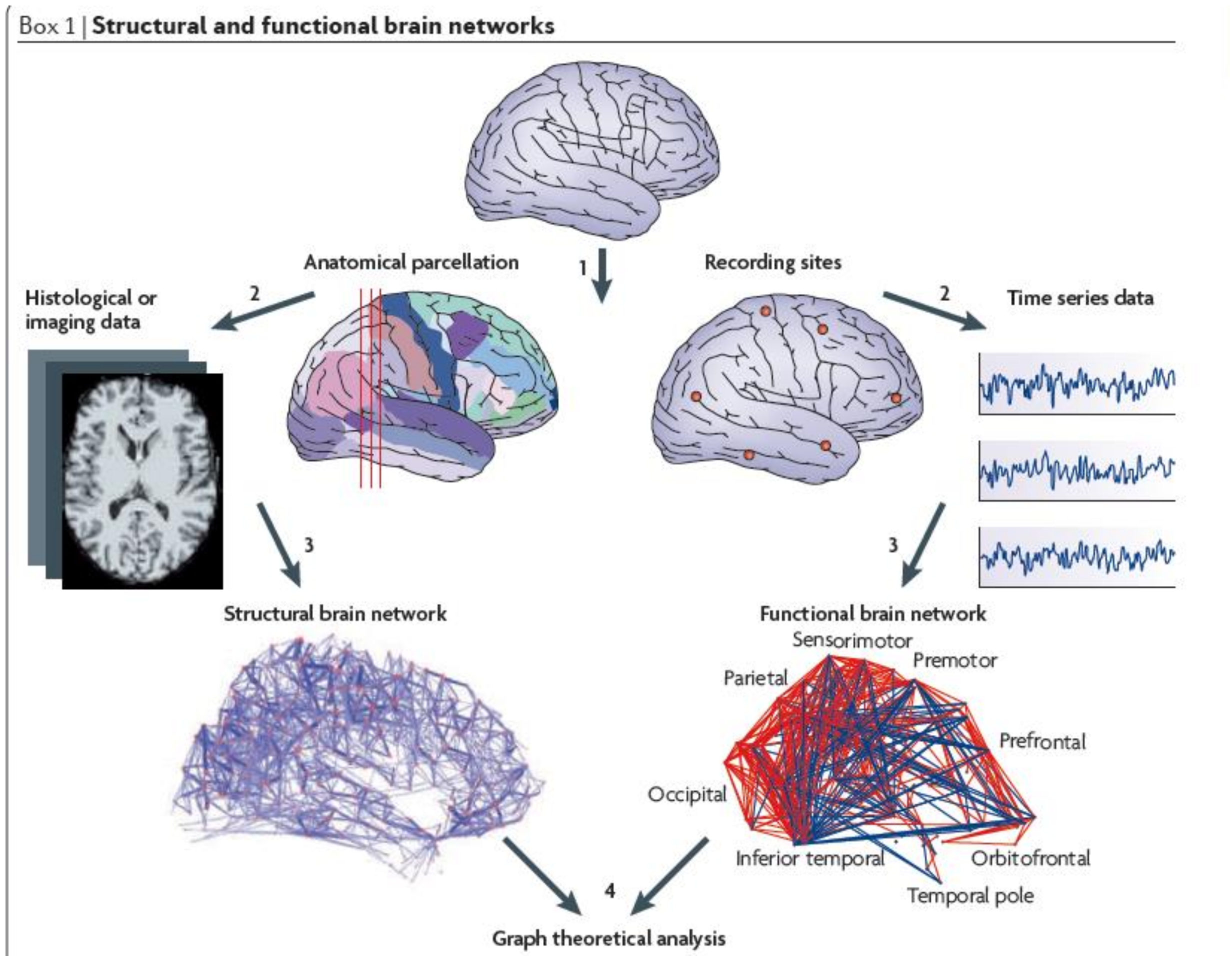
How are we going to define connectivity?
How are functional, structural, and effective connectivity related?

Networks

- Random
- Ordered
- Scale-free
- Small-world

- How do clustering coefficients and path lengths vary between functional and structural analyses?

Our challenge:



Ideas:

- Structural connections of the human brain
  - Sex differences
  - Epilepsy, seizures
  - Brain damage/impairment
  - Memory and higher cognition

Sources:

- Stam, Cornelis, and Jaap Reijneveld. “Graph theoretical analysis of complex networks in the brain.” *Nonlinear Biomedical Physics* 1:3. n. pag. Print.
- Bullmore, Ed, and Olaf Sporns. “Complex brain networks: graph theoretical analysis of structural and functional systems.” Nature 1 Mar. 2009:186-198. Web.