Competition Graphs and Permutation Patterns

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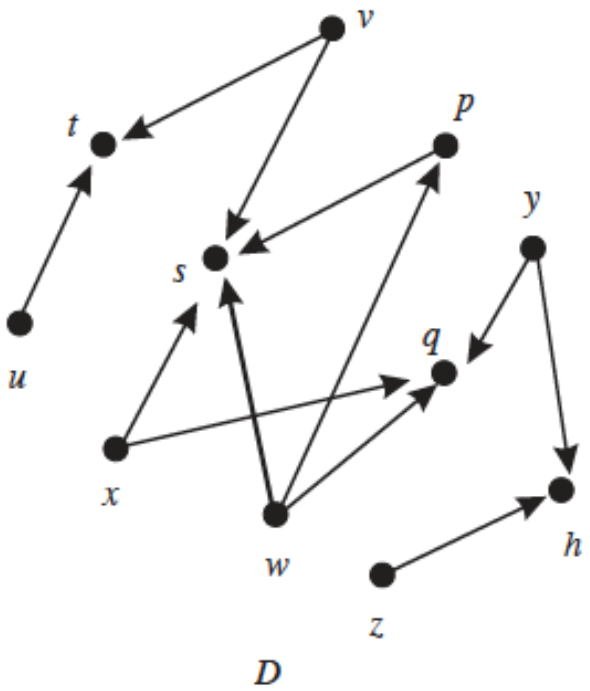
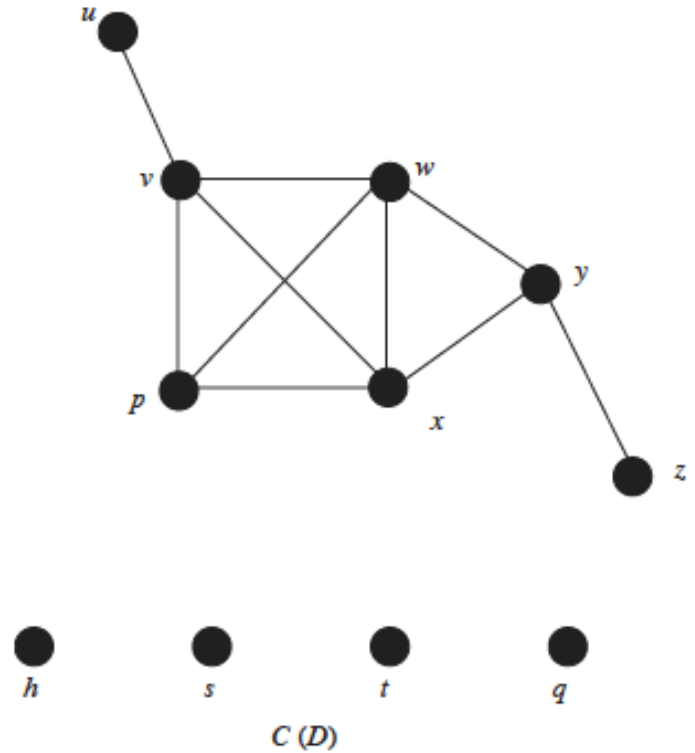
Overview

Competition Graphs

- A **digraph** is a set of vertices $V(D)$ and a set of arcs $A(D)$. An arc (u, v) is represented by an arrow from vertex u to vertex v.
- A **competition graph** of a digraph D has the same vertex set $V(D)$ and an edge uv if D contains arcs (u, w) and (v, w).
  - In short, the edge uv is in the competition graph if u and v "compete" over another vertex (in this case, w)
- We use the notation $C(D)$ to denote the competition graph of digraph D.

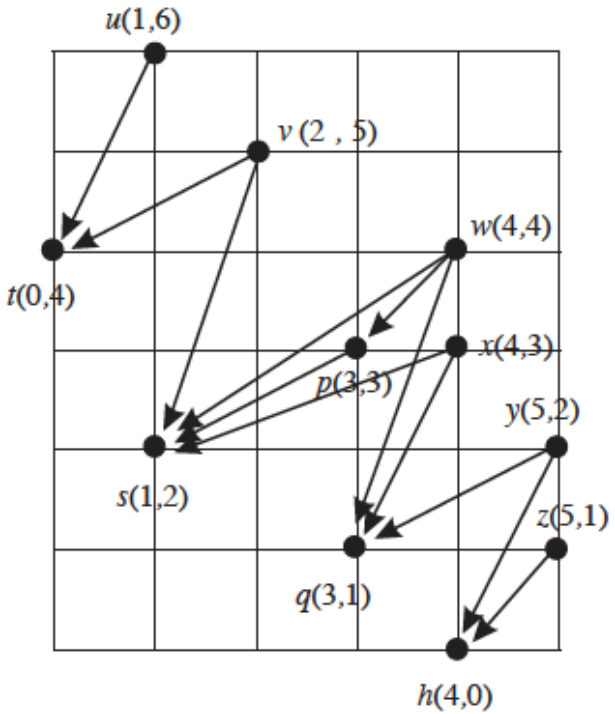
Example of a Competition Graph





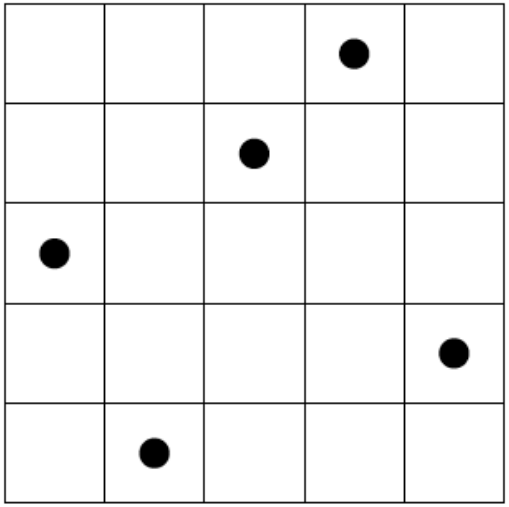
Doubly Partial Orders

- A **doubly partial order on S** is a set of objects S and a relation $\prec$.
- In the context of competition graphs, we mainly consider doubly partial orders in $\mathbb{R}^2$.
  - Let our set $S = \{(x_1, y_1), (x_2, y_2), \ldots, (x_k, y_k)\}$.
  - We say that $(x_i, y_i) \prec (x_j, y_j)$ if both $x_i < x_j$ and $y_i < y_j$.
- The digraph D induced by S has vertex set S and an arc from (x_j, y_j) to (x_i, y_i) iff $(x_i, y_i) \prec (x_j, y_j)$.



Permutations

- A **permutation** is a sequence of numbers $\pi = \pi_1\pi_2 \ldots \pi_n$, where each $\pi_i \in \{1, 2, \ldots, n\}$.
  - For any given n, we have $n!$ distinct permutations.
  - The set of all permutations on n elements is denoted as S_n.
- Below is one way to visualize a permutation. Here, we plot (i, π_i) for $\pi = 3\,1\,4\,5\,2$:



Popular Problems in Competition Graph Theory

In the past couple of decades, researchers have been interested in a few main classes of problems pertaining to competition graphs:

- Computing competition numbers of certain classes of graphs
- Variants of competition graphs (common enemy graphs, domination graphs, m-step graphs)
- **Finding competition graphs for certain classes of digraphs**

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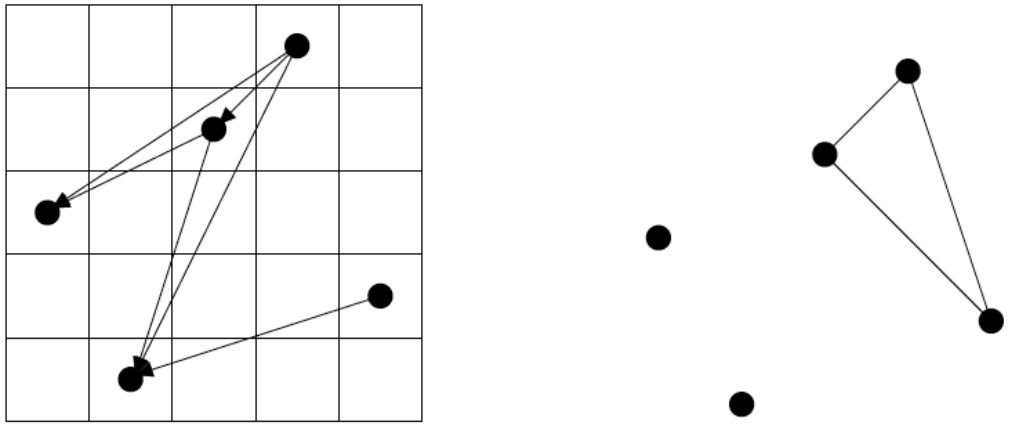
- Characterize the competition graphs of induced digraphs on permutations.
  - For instance, for $\pi = 3\,1\,4\,5\,2$:



- Characterize the competition graphs of induced digraphs on special classes of permutations
  - i.e. permutations that avoid a certain pattern, permutations that contain a certain pattern
  - Hopefully, this will also show us new insights about permutation patterns in general!

References

- H. H. Cho, S. R. Kim, A class of acyclic digraphs with interval competition graphs, Discrete Appl. Math. 148 (2005) 171-180
- S. Kitaev, T. Mansour, A survey on certain pattern problems, 2000 Mathematics Subject Classification: Primary 05A05, 05A15; Secondary 30B70, 42C05

Thank You!