The Yang-Mills Flow on Four-Manifolds with Tubular Ends

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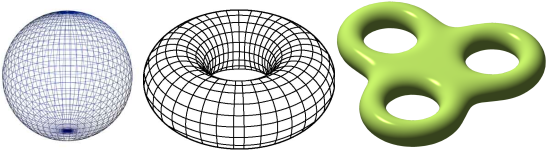
What is a manifold?

Definition

A **manifold** of dimension n (or an n-manifold) is a space that locally looks like normal Euclidean space $\mathbb{R}^n$.

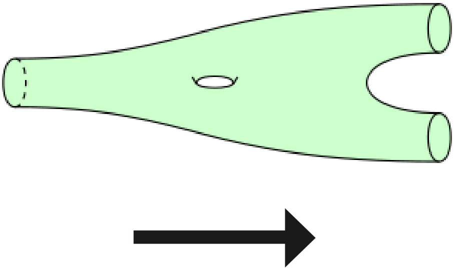
- Examples (in two dimensions):



- Three and four-manifolds are harder to visualize.

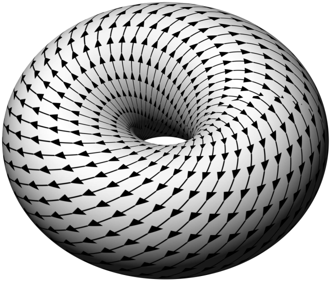
Manifolds with tubular ends

- Manifolds which “go off to infinity.”



- Can think of as "space-time," with cross sections as "space."

Gauge Fields



- Generalize electromagnetic fields.
- May have multiple different types of charges.
- Ex: Strong nuclear force, Weak nuclear force.

Instantons

- Yang-Mills energy, $\mathcal{YM} : \{\text{Gauge Fields}\} \to [0, \infty)$

Definition

A gauge field is an **instanton** if it has the minimum possible Yang-Mills energy. On a manifold with tubular ends, a gauge field is an instanton if it has minimal Yang-Mills energy for fixed boundary conditions "at infinity."

- Can think of instanton as giving a way for the field to propagate from the distant past to the distant future.

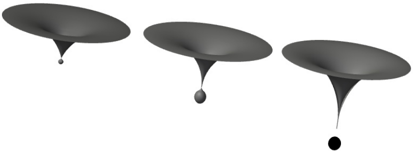
Yang-Mills flow

- How do we find instantons?
- Idea: start with arbitrary field A_0 on space-time, and then let field evolve to decrease energy as fast as possible.

$$\frac{\partial A_s}{\partial s} = -\nabla(\mathcal{YM}(A_s)).$$

- LHS: Change in field with "time" s.
- RHS: Negative gradient of Yang-Mills energy.

Issues



- Flow may develop singularities.
- "Bubbling": field concentrates in a single region with infinite density.
- Need to eliminate or control bubbling.

State of the art

- In two and three dimensions, flow well understood and controlled.
- Same in special cases for four dimensions (E.g. with a "holomorphic structure").
- Four dimensions with tubular ends not resolved.

What would this get us?

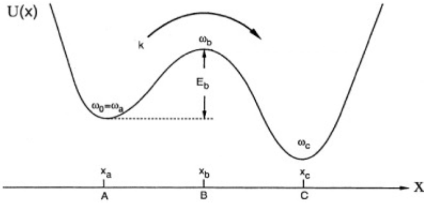
- Help us locate instantons.
- Allow us to understand “approximate instantons” - fields with small, but not quite minimal energy.
- Tool to turn approximate instantons into real, exact instantons.

Applications to geometry

- The Atiyah-Floer conjecture: $HF_{\text{instantons}} = HF_{\text{symplectic}}$?
- Geomeric invariant theory: algebraic conditions for understanding when instantons exist.
- Geometric heat flows: Ricci flow, harmonic map flow, etc.

Applications to real life

- Standard Model of particle physics is a quantum Yang-Mills theory.
- Instantons allow us to calculate tunneling probabilities.



Questions?

$$\frac{\partial A_s}{\partial s} = -\nabla(\mathcal{YM}(A_s))$$