$\mathcal{IV}$-matching

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Introduction and results

- Introduction
  - Article by Fiala, Klavík, Kratochvíl and Nedela [1]
  - Regular graph covers
  - Subproblem reducing to $\mathcal{IV}$-MATCHING
- Our results
  - $\mathcal{IV}$-MATCHING is $\mathcal{NP}$-complete
  - Only 2-vertex-connected instances are interesting
  - We identified some solvable instances

Definition – Instance

- Graph $G = (V, E)$, $V = V_1 \dot\cup \cdots \dot\cup V_\ell$
- Layers split into clusters
- Edges of G described by macroedges on clusters
- Macroedge between two clusters $\Rightarrow$ a complete bipartite subgraph
- Macroedges between V_{2k} and V_{2k+1} form a matching
- Macroedges between V_{2k-1} and V_{2k} form an arbitrary graph

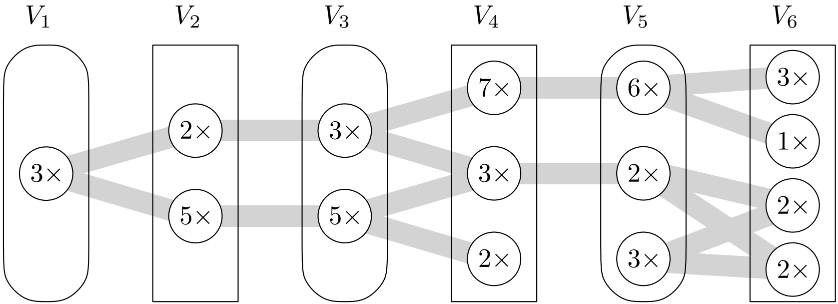

Figure 1: Graph on clusters

Definition – $\mathcal{IV}$-matching

- All vertices must be covered
- Between V_{2k} and V_{2k+1}: $\mathcal{V}$s – paths of the length two with centers in the layer V_{2k+1}
- Between V_{2k-1} and V_{2k}: $\mathcal{I}$s – edges

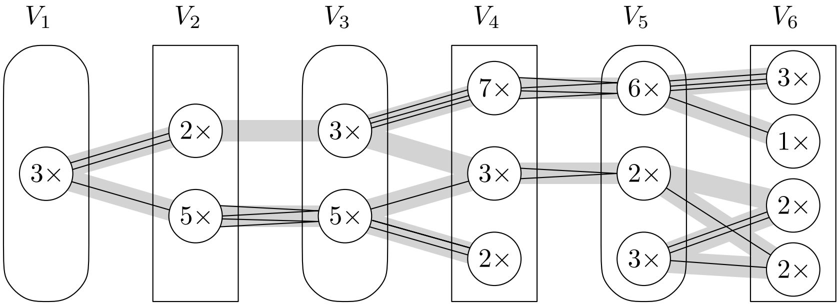

Figure 2: An example of $\mathcal{IV}$-matching

3D-matching

- Instance: Hypergraph with equally sized partites X, Y, Z and the set of edges $E \subseteq X \times Y \times Z$
- Question: Is there a perfect matching in the hypergraph?
- $\mathcal{NP}$-complete: number 17 in Karp's set of 21 problems [2]

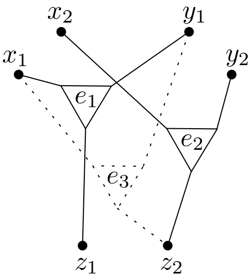

Figure 3: 3D-MATCHING

NP-completeness

- The reduction

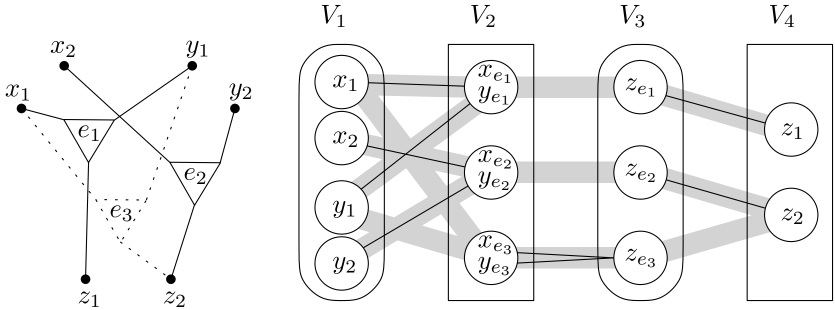

Figure 4: An instance of 3D-MATCHING with a perfect matching and a corresponding $\mathcal{IV}$-MATCHING instance with an $\mathcal{IV}$-matching.

Other results

- Only 2-vertex-connected instances – cut-vertices may be eliminated
- Instances with a constant number of clusters are solvable in polynomial time
- $\mathcal{IV}$-matching on a cycle can be solved in linear time

References

[1] J. Fiala, P. Klavík, J. Kratochvíl, and R. Nedela, *Algorithmic aspects of regular graph covers with applications to planar graphs*, in Automata, Languages, and Programming, vol. 8572 of Lecture Notes in Computer Science, Springer Berlin Heidelberg, 2014, pp. 489–501.

[2] R. Karp, *Reducibility among combinatorial problems*, in Complexity of Computer Computations, The IBM Research Symposia Series, Springer US, 1972, pp. 85–103.