L-bounded flows and cuts

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Definition

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L-bounded cut is a set of edge F, such that network $((V, E \setminus F), s, t, c)$ does not contain any s-t path of length L or smaller.

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Examples:

- Imagine lab, where scientists work with fast degrading liquid. And the liquid is transported in pipes. We need to transport the liquid as fast as possible.
- Take streets in a city as a network. And we want to find the best location for fire stations. We can take L-bounded flow problem to determine, how fast the fire brigade can be at the place of fire.

Computational complexity

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- L-bounded cut is NP-hard for $4 \leq L \leq n^{1-\epsilon}$ – even constant approximation is NP-hard.

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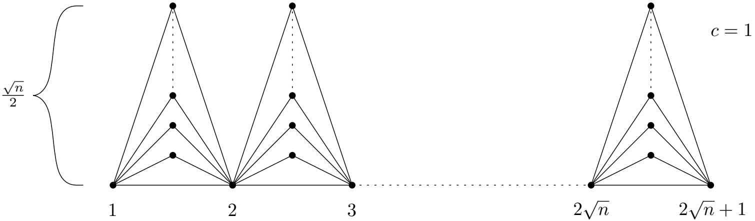

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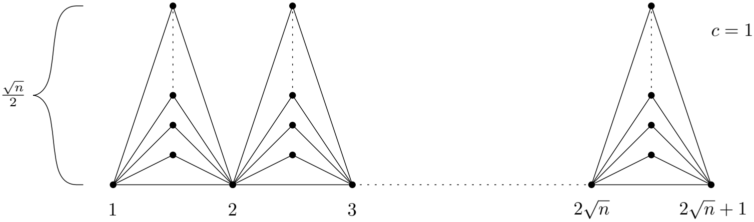

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- The L-bounded flow has size 2.

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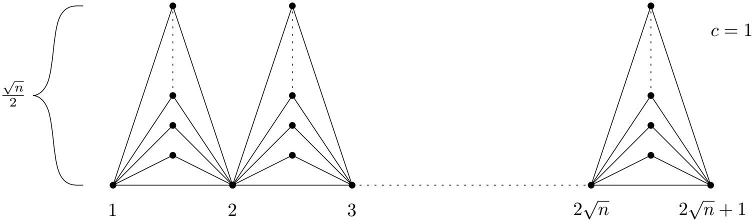

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- The L-bounded flow has size 2.
- The L-bounded cut has size $\Omega(\sqrt{n})$.

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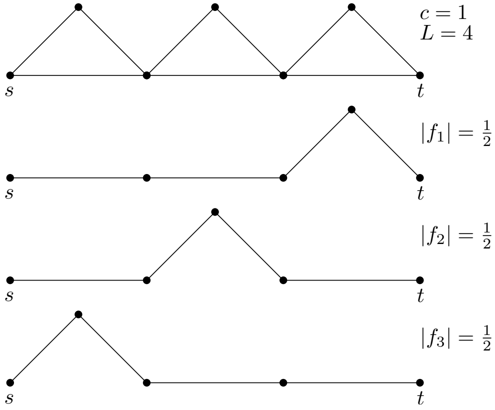

Open problems

We know the $\mathcal{O}(L^{tw} n)$-algorithm for L-bounded cut.

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Problem

Is there an algorithm, which runs in time independently on L?