Graph Theory and Some Applications

Shannon Jordan
Morgan State University

Roger Licairac
Rutgers University

Aquia Richburg
Morehouse College

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Definitions

- A **graph** G is a triple consisting of a **vertex** set $V(G)$, an **edge** set $E(G)$ and a relation that associates each edge with two vertices.
- A **subgraph** of G is a graph H where $V(H) \subseteq V(G)$, $E(H) \subseteq E(G)$ and the edge vertex assignments are the same
- A **directed graph** is a graph where each edge has an arrow pointing to a vertex.
- The endpoints of a directed edge are called the **head** and the **tail**. A directed edge points from the tail to the head.

Examples

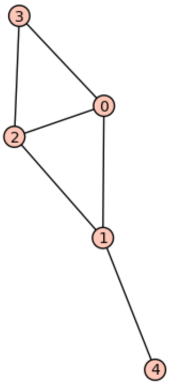


Figure : A simple graph on 5 vertices

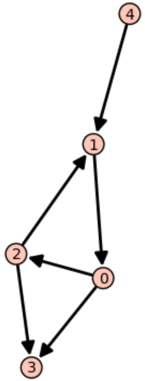


Figure : A simple digraph on 5 vertices

Definitions

- The **connectivity** $K(G)$ of a connected graph G is the minimum number of vertices whose removal disconnects G.
- A **cut-vertex** is a single vertex whose removal disconnects a graph.

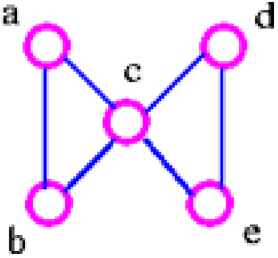


Figure : A graph with $K(G) = 1$

Defintions

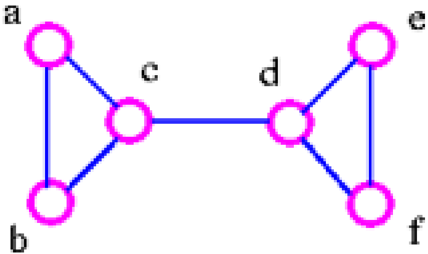


Figure : A graph with $\lambda(G) = 1$

- The **edge connectivity** $\lambda(G)$ of a connected graph G is the minimum number of edges whose removal disconnects G.
- A **bridge** is a single edge whose removal disconnects a graph.
- A **cut set** of a connected graph G is a set S of edges with the following properties:
    - The removal of all edges in S disconnects G but the removal of some edges does not disconnect G.

Definitions

- A **competition graph** is constructed by using the same vertex set as a digraph and placing an edge between two vertices if the directed edges from those vertices have the same tail
- A graph is an **interval graph** if there exists intervals on the real line such that two vertices are connected by an edge if and only if their corresponding intervals overlap.

Consider an ecosystem where you treat each vertex as a species and an edge from x to y exists if x preys on y.

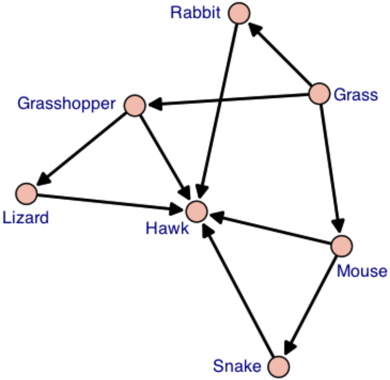


Figure : A simple food web

Examples

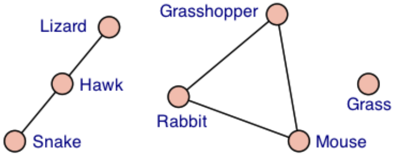
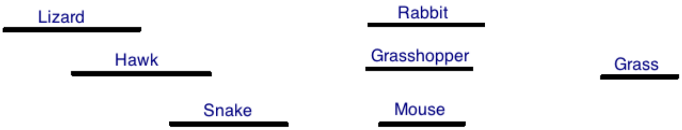


Figure : Above: The competition graph derived from a simple food web; Below: Shows that this competition graph is an interval graph

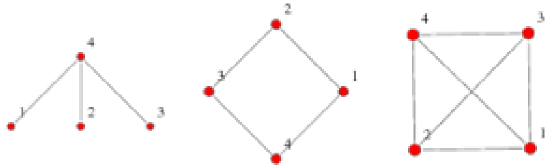


Problems

1. What characteristics of a digraph guarantee that its competition graph is an interval graph?
2. Let $V \subset \mathbb{R}^n$ be finite. Let D be a digraph such that $V(D) = V$ and there is an edge from $(a_1, a_2, \ldots, a_n)$ to $(b_1, b_2, \ldots, b_n)$ if and only if $a_i > b_i$ for all i. What are forbidden subgraphs, if any, of the competition graph $C(D)$ of D?

Definitions

- The **adjacency matrix** of a graph on n vertices is an $n \times n$ matrix $A = (a_{i,j})$ in which the entry $a_{i,j} = 1$ if there is an edge from vertex i to vertex j and is 0 otherwise.

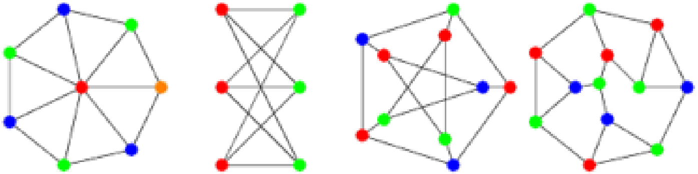


$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

Figure : Three graphs and their respective adjacency matrices

Definitions

- A **vertex coloring** is an assignment of labels or colors to each vertex of a graph such that no edge connects two identically colored vertices.
- The most common type of vertex coloring seeks to minimize the number of colors for a given graph. Such a coloring is known as the **minimum vertex coloring**.



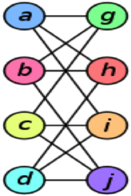
Definitions

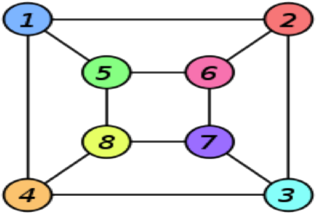
- An **isomorphism** of graphs G and H is a bijection f between $V(G)$ and $V(H)$ so that any two vertices x and y are adjacent in G if and only if $f(x)$ and $f(y)$ are adjacent in H.
- This is known as an edge-preserving bijection.
- If an isomorphism exists between two graphs, then the graphs are isomorphic.

Examples

Although these two graphs look different they are isomorphic. The isomorphism between the two graphs is:

$f(a) = 1$
$f(b) = 6$
$f(c) = 8$
$f(d) = 3$
$f(g) = 5$
$f(h) = 2$
$f(i) = 4$
$f(j) = 7$





Fingerprint Background

- There are three main types of finger prints: visible prints, latent prints and impressed prints.
- Visible prints are left in medium, like blood and reveals them to the naked eye.
- Latent prints are not apparent to the naked eye. They are formed from sweat, glands on the body or water, salt and amino acids. They become visible by dusting, fuming or chemical reagents.
- Impressed prints are left in soft pliable surfaces like clay or wax. They become visible when viewed or photographed without development.

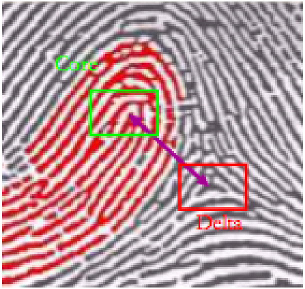
Three Levels of Ridge Features

- Level One Detail - General Ridge Flow
  - Class characteristics only - No individualizing detail
  - Fingerprint patterns and ridge flow (shape)
- Level Two Detail - Individual Ridge Path
  - Major ridge features
  - Ridge endings, bifurcations, etc., their positions and relationship to other features
  - Allows some individualization
- Level Three Detail - Individual Ridge Appearance
  - High level of detail
  - Smallest features (pore and ridge structure) are visible for comparison
  - The most individualizing detail

Fingerprint Classification

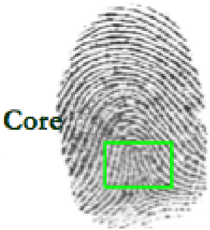
- Pattern Types
  - Arch - plain or tented
  - Loop - radical or ulnar (depends on which side the head and the tail points towards)
  - Whorl - plain, accidental, double loop, central pocket
  - Branch
- Size of the patterns
- Position of the patterns on the finger
- Ridge characterization
  - Ridge dot - an isolated ridge unit whose length approximates its width
  - Ending ridge - a single friction ridge that terminated within the friction ridge structure
  - Ridge crossing - a point where two ridge units intersect
  - Bridges - a connecting friction ridge between parallel running ridges

Loops: Radial and Ulnar



- Loops have one core, one delta and the ridge count
- There are two types of loops: radial and ulnar
    - Radial loops slant towards the radial (thumb) side of the hand.
    - Ulnar loops slant towards the ulnar (pinky finger) side of the hand.

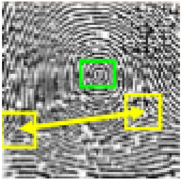
Arches: Plain and Tented



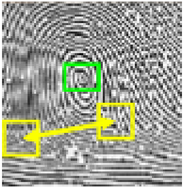
- Arches are classified by having no delta and the ridge count.
- It is possible for the top of the arch to depict what we call a core.
- Loops are of two types: plain and tented
  - Plain arches have no core and no delta
  - Tented arches are "peaked" in the center with what would be considered a core.

Whorls: Plain, Accidental, Double loop, Central pocket

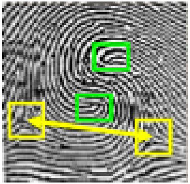
- Whorls are characterized by having two or more deltas and the ridge count.
- Whorls are categorized in four different parts: plain, accidental, double loop and central pocket



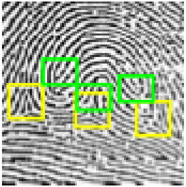
Plain



Central pocket



Double loop



Accidental

Graph Theory and Fingerprints