Forbidden Subgraphs of Competition Graphs

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Outline

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2. Competition Graphs
3. Partial Order
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Graph Theory Basics

- A **graph** G is a triple consisting of a **vertex** set $V(G)$, an **edge** set $E(G)$ and a relation that associates each edge with two vertices.
- A **subgraph** of G is a graph H where $V(H) \subseteq V(G)$, $E(H) \subseteq E(G)$ and the edge vertex assignments are the same
- A **directed graph** is a graph where each edge has an arrow pointing to a vertex.
- The endpoints of a directed edge are called the **head** and the **tail**. A directed edge points from the tail to the head.
- A graph H is **forbidden** in G if it is not isomorphic to any subgraph of G.

Examples

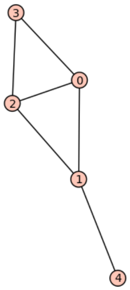


Figure : A simple graph on 5 vertices

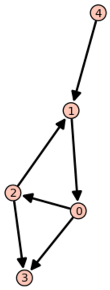


Figure : A simple digraph on 5 vertices

Competition Graphs

- A **competition graph** $C(D)$ is constructed from a digraph D having the same vertex set as D and an edge between two vertices x and y exist if there exists a vertex w such that $(w, x), (w, y) \in A(D)$.
- A graph is an **interval graph** if there exists intervals on the real line such that two vertices are connected by an edge if and only if their corresponding intervals overlap.
- The **boxicity** of G is the smallest p such that we can assign to each vertex of G a box in Euclidean p-space so that two vertices are neighbors if and only if their boxes overlap.

Example

Consider an ecosystem where each vertex is a species and an edge from x to y exists if x preys on y.

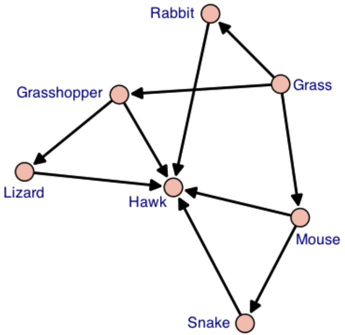


Figure : A simple food web

Example

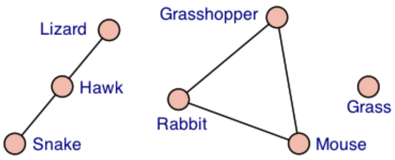
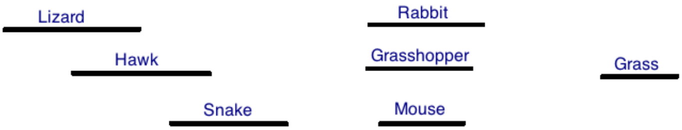


Figure : Above: The competition graph derived from a simple food web; Below: Shows that this competition graph is an interval graph



Partial Order

Definition

A **partial order** is a binary relation over a set S which is reflexive, antisymmetric and transitive.
A **partially ordered set**, or poset, is a set with a partial order.

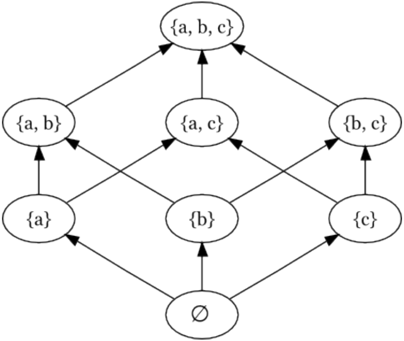
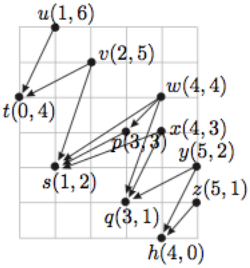


Figure : A Hasse diagram on a set of size 3

Partial Order



D

Figure : A doubly partial order D and a doubly partial order on a subset of $\mathbb{R}^2$ which is isomorphic to D

Definition

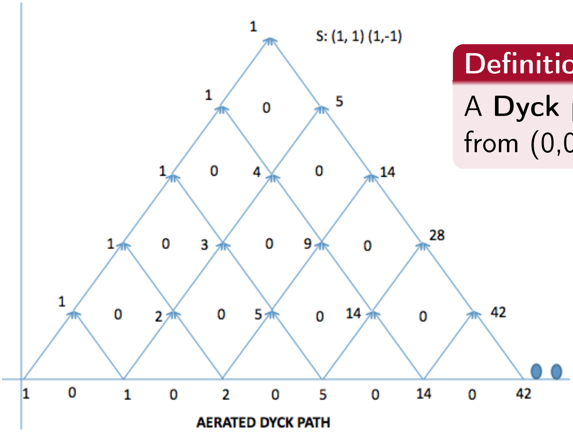
A relation $\swarrow$ on a subset S of $\mathbb{R}^2$ is a **doubly partial order on** S if $(x, y) \swarrow (z, w)$ whenever $x < z$ and $y < w$ for $(x, y), (z, w) \in S$. A double partial ordered set is a set with a doubly partial order. A digraph D is called a **doubly partial order** if D is isomorphic to a digraph of a doubly partial order relation $\swarrow$ on a subset of $\mathbb{R}^2$.

Partial Order

Definition (Choi, Kim, Kim, Lee 2014)

A relation $\swarrow$ on a subset S of $\mathbb{R}^n$ is an **n-tuply partial order on S** if, for $(x_1, x_2, \ldots, x_n), (y_1, y_2, \ldots, y_n) \in S$, $(x_1, x_2, \ldots, x_n) \swarrow (y_1, y_2, \ldots, y_n)$ whenever $x_i < y_i$ for $i = 1, \ldots, n$. For an integer $n \geq 2$, a digraph D is called an **n-tuply partial order** if D is isomorphic to a digraph of an n-tuply partial order relation $\swarrow$ on a subset of $\mathbb{R}^n$. A graph G is the **competition graph of an n-tuply partial order** if there is an n-tuply partial order D such that G is isomorphic to the competition graph of D.

Dyck Path



Definition

A **Dyck path** is a staircase walk from (0,0) to (n,n).

Previous Results

Theorem (Cho, Kim 2005)

Competition graphs of a digraph of doubly partial order are interval graphs.

Theorem (Cho, Kim 2005)

An interval graph with sufficiently many isolated vertices is the competition graph of a doubly partial order.

Previous Results

Theorem (Choi, Kim, Kim, Lee 2014)

Every interval graph can be made into the competition graph of a triply partial order by adding sufficiently many isolated vertices.

Theorem (Choi, Kim, Kim, Lee 2014)

Every tree can be made into the competition graph of a triply partial order by adding sufficiently many isolated vertices.

Corollary (Choi, Kim, Kim, Lee 2014)

If a graph G is the competition graph of a triply partial order, then G does not contain $K_{3,3}$ as an induced subgraph.

The Problem

Let D be a digraph of n-tuply partial order. What are forbidden subgraphs, if any, of the competition graph $C(D)$ of D?

Results

Theorem

If loops are understood a digraph D of n-tuply partial order represents a poset.

Theorem

Given a digraph D of n-tuply partial order a forbidden subgraph of $C(D)$ is C_m where $m \geq 4$.

Theorem

Given a digraph D of n-tuply partial order any subgraph with maximum degree $n-1$ is forbidden in $C(D)$.

Future Work

- Continue characterizing forbidden subgraphs in competition graphs
- Generalize to competition graphs constructed from digraphs isomorphic to general posets

תודה

Dankie Gracias

شكراً

Спасибо Merci Takk

Köszönjük Terima kasih

Grazie Dziękujemy Dėkojame

Ďakujeme Vielen Dank Paldies

Kiitos Täname teid 谢谢

Thank You

Tak

感謝您 Obrigado Teşekkür Ederiz

감사합니다

Σας Ευχαριστούμ ขอบคุณ

Bedankt Děkujeme vám

ありがとうございます

Tack