

$L(p, q)$ - Labeling

Channel Assignment Problem

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Graph Coloring

- $G(V,E)$
- vertex coloring $c : V \rightarrow \{1, 2, \dots, k\}$
- $\{u, v\} \in E : c(u) \neq c(v)$

Chromatic number

What is the minimal number of colors $\chi(G)$ so we can color vertices of a given graph properly?

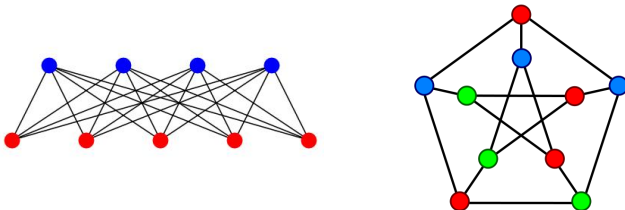


Figure 1 : Source: web.math.princeton.edu, wikipedia.org

Edge Coloring

- edge coloring $c : E \rightarrow \{1, 2, \dots, k\}$
- $\{u, v\}, \{v, w\} \in E : c(\{u, v\}) \neq c(\{v, w\})$

Chromatic index

What is the minimal number of colors $\chi'(G)$ so we can color edges of a given graph properly?

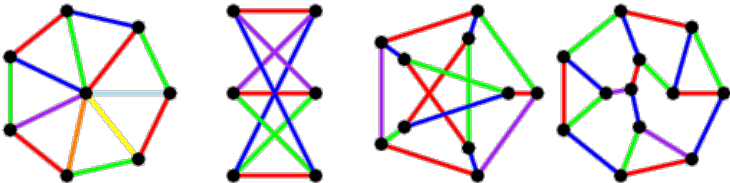


Figure 2 : Source: wolfram.com

Channel Assignment Problem



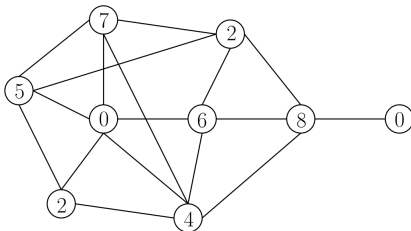
Figure 3 : Source: hbl.cz, ct24, liberec-me-bavi.cz

$L(p, q)$ - Labeling definition

- labeling $l : V \rightarrow \{1, 2, \dots, k\}$
- $\{u, v\} \in E : |l(u) - l(v)| \geq p$
- $\{u, v\}, \{v, w\} \in E : |l(u) - l(w)| \geq q$

Span

What is the minimal label $\lambda(G)$ so we can color vertices of a given graph properly?



$L(1, 1)$ - Labeling

$L(1, 1)$ - Labeling of G = Coloring of G^2

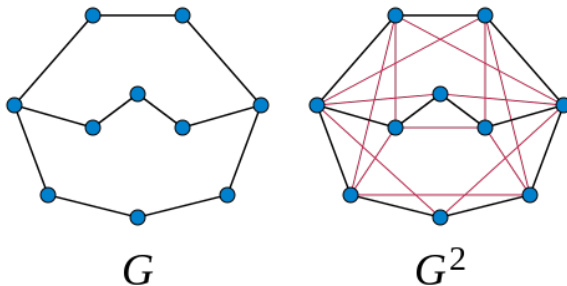


Figure 4 : Source: David Eppstein

$L(2, 1)$ - Labeling On Interval Graphs

- The most studied case of $L(p, q)$
- Known for e.g. trees, cycles, chordal graphs
- Interval graphs - known for **unit interval graphs only**

Our Objective

Find a tight bound for span of $L(2, 1)$ on interval graphs.

Other open problems

- Other intersection graphs (boxes, circles...)
- Vizing's theorem about edge coloring for $L(2, 1)$ - Labeling
- Other p, q values