Challenges of Identifying Integer Sequences

(Browsing the OEIS)

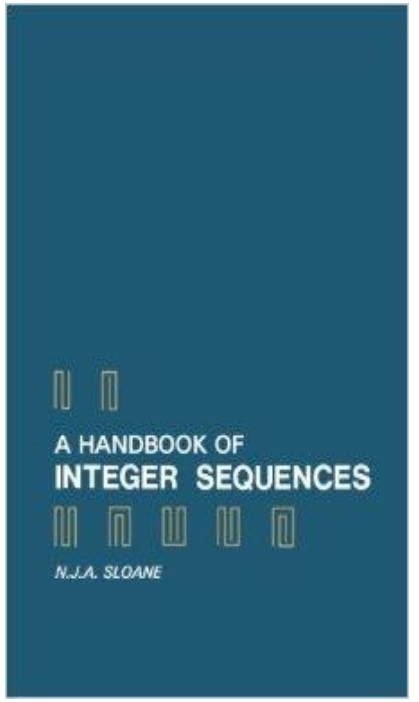
Pratik Koirala (Howard University)

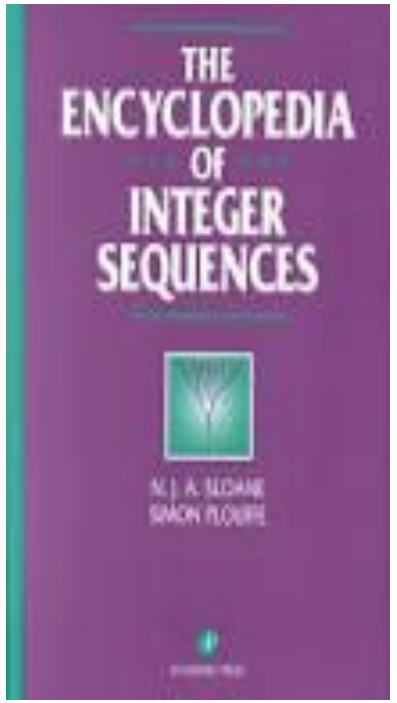
Otis Tweneboah (Texas Southern University)

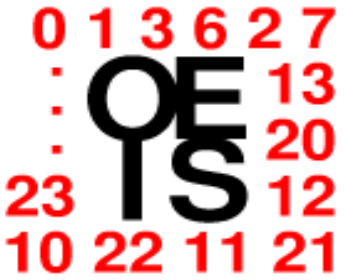
Mentors: Dr. Eugene Fiorini, Nathan Fox, Dr. Brian Nakamura

The On-Line Encyclopedia of Integer Sequence

- Dr. Neil Sloane
- Punch cards, 1964
- A Handbook of Integer sequences, 1973
- The Encyclopedia of Integer Sequences, 1995
- OEIS website (www.oeis.org), 1996
- Over 250,000 sequences, 2015

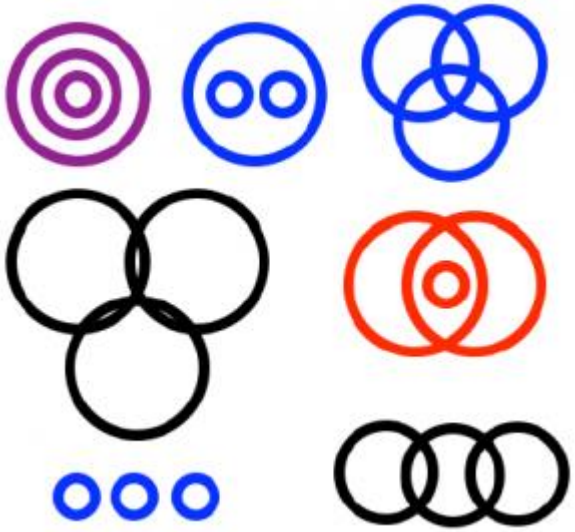


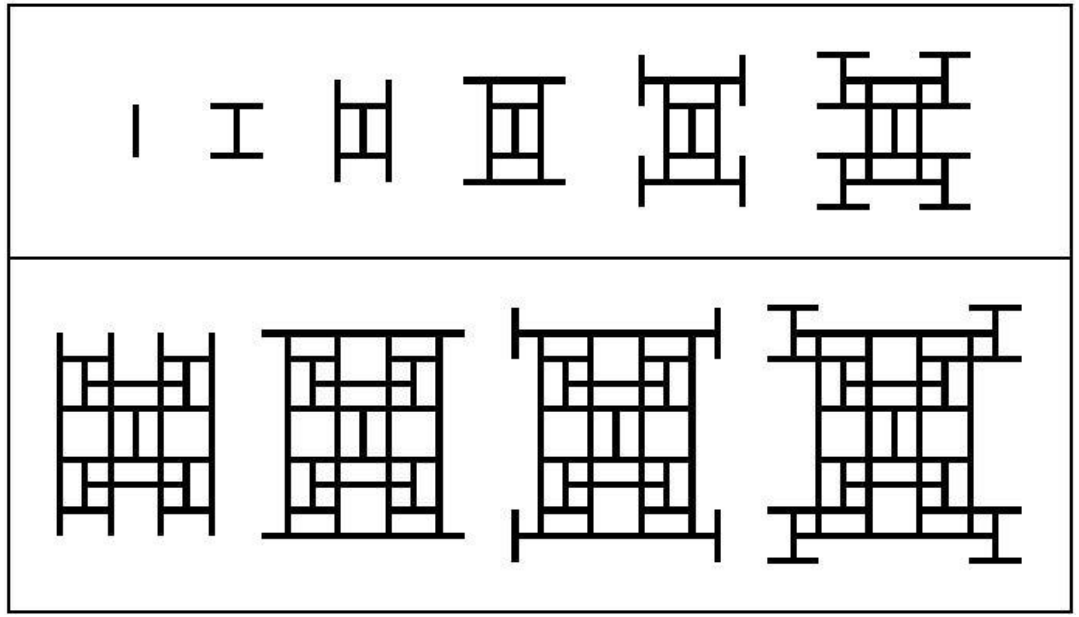




Browsing the OEIS

- Search: By terms, sequence number, keywords
- Keywords : more, hard, easy, new, etc.
- Features: Language, music, graphs
- Contribution : OEIS account, Approval





Sequences that we worked on

- Recamán's Sequence (A005132)
- A105403 : (Variations - A259558, A259559, A259562, A259564)
- A029790
- A248872

Recamán's Sequence (A005132)

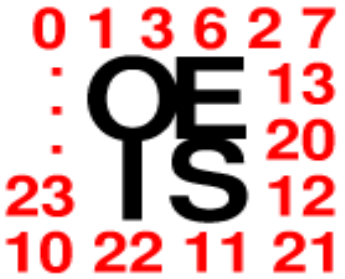
- Recamán's sequence is a mathematical sequence,

defined as such: $a_0 = 0$

$$a_n = \begin{cases} a_{n-1} - n & if\ (a_{n-1} - n) > 0\ and\ not\ already\ in\ the\ sequence \\ a_{n-1} + n & otherwise, \end{cases}$$

n	0	1	2	3	4	5	6	7
a_n	0	1	3	6	2	7	13	20

- Python Scripts
  - Recaman(N) : Produced the Recaman's sequence for *n* from 0 to N.
  - Recaman(m, N) : Produced the Recaman's sequence with the initial value a_0 = m and for *n* from 0 to N.

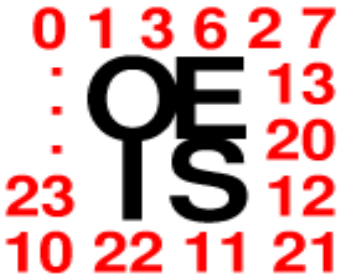


Sequence A105403

- Numbers n such that prime(n) – 1 and prime(n+1) – 1 have the same largest prime factor.
    - Currently known terms in the sequence : 2, 30, 53, 217
- Example: when n = 2,
    - prime(2) – 1 = **3** – 1 = **2** = 1 * **2**
    - prime(3) – 1 = **5** – 1 = **4** = 2 * **2**

  when n = 30,
    - prime(30) – 1 = **113** – 1 = **112** = 2 * 2 * 2 * 2 * **7**
    - prime(31) – 1 = **127** – 1 = **126** = 2 * 3 * 3 * **7**

  when n = 15,
    - prime(15) – 1 = **47** – 1 = **46** = 2 * **23**
    - prime(16) – 1 = **53** – 1 = **52** = 2 * 2 * **13**



Contributions

- No more terms < 5760000. - David Wasserman, May 02 2008
- No more terms < 11078936. - Otis Tweneboah, Pratik Koirala, Eugene Fiorini, Nathan Fox, Jun 30 2015
- No more terms < 26355866. - Otis Tweneboah, Pratik Koirala, Eugene Fiorini, Nathan Fox

A105403 variation(1) – Sequence A259558

- Numbers n such that prime(n) – 1 and prime(n+1) – 1 have the same number of distinct prime factors.
  - Terms in the sequence: 2, 4, 5, 8, 9, 12, 15, 16, 18, 19, 23, 24, 25, 28, 29, 31, 36, 38….
- Example: when n = 5,
  - prime(5) – 1 = **11** – 1 = **10** = **2** * **5** ➔ 2 distinct prime factors
  - prime(6) – 1 = **13** – 1 = **12** = 2 * **2** * **3** ➔ 2 distinct prime factors

  when n = 10,
  - prime(10) – 1 = **29** – 1 = **28** = 2 * **2** * **7** ➔ 2 distinct prime factors
  - prime(11) – 1 = **31** – 1 = **30** = **2** * **3** * **5** ➔ 3 distinct prime factors

A105403 variation(2) – Sequence A259559

- Numbers n such that prime(n) – 1 and prime(n+1) – 1 have the same number of prime factors, including repeats.
  - Terms in the sequence: 3, 4, 10, 12, 19, 29, 34, 36, 45, 46, 50, 61, 85, 89, 91, 104, 112, 117…..
- Example: when n = 5,
  - prime(5) – 1 = **11** – 1 = **10** = **2** * **5** ➔ No. of prime factors = 2
  - prime(6) – 1 = **13** – 1 = **12** = **2** * **2** * **3** ➔ No. of prime factors = 3

  when n = 10,
  - prime(10) – 1 = **29** – 1 = **28** = **2** * **2** * **7** ➔ No. of prime factors = 3
  - prime(11) – 1 = **31** – 1 = **30** = **2** * **3** * **5** ➔ No. of prime factors = 3

A105403 variation(3) – Sequence A259562

- Numbers n such that the sum of the distinct prime factors of prime(n) – 1 and prime(n+1) – 1 are the same.
  - Terms in the sequence: 2, 414, 556, 3962, 4972, 6151, 6521, 8440, 8665, 13769, 13909, 15576, 16696, 17176, 19926, 20630….
- Example:
  - when n = 53:

    prime(53) – 1 = **241** - 1 = 240 = 2 * 2 * **2** * **3** * **5** ➔ Sum = 10

    prime(54) – 1 = **251** – 1 = 250 = **2** * **5** * 5 * 5 ➔ Sum = 7
  - when n = 414:

    prime(414) – 1 = **2851** – 1 = 2850 = **2** * **3** * **5** * 5 * **19** ➔ Sum = 29

    prime(415) – 1 = **2857** – 1 = 2856 = 2 * 2 * **2** * **3** * **7** * **17** ➔ Sum = 29

A105403 variation(4) – Sequence A259564

- Numbers n such that the sum of the prime factors (including repeats) of prime(n) – 1 and prime(n+1) – 1 are the same.
  - Terms in the sequence: 5, 7, 11, 30, 133, 160, 415, 527, 883, 1257, 2025, 2771, 2775, 6650, 6932, 13793, 19091, 30695, 32341….
- Example:
  - when n = 30:

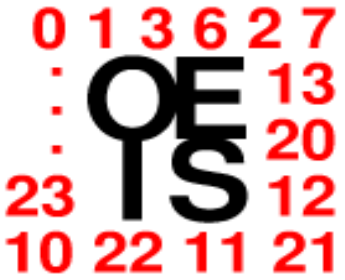
    prime(30) – 1 = **113** – 1 = 112 = **2 * 2 * 2 * 2 * 7** ➔ Sum = 15

    prime(31) – 1 = **127** – 1 = 126 = **2 * 3 * 3 * 7** ➔ Sum = 15

  - when n = 53:

    prime(53) – 1 = **241** - 1 = 240 = **2 * 2 * 2 * 3 * 5** ➔ Sum = 14

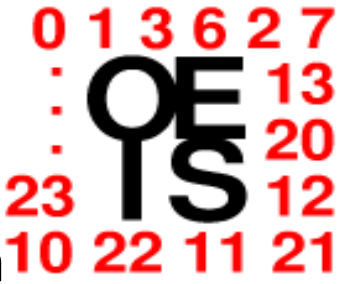
    prime(54) – 1 = **251** – 1 = 250 = **2 * 5 * 5 * 5** ➔ Sum = 17



Sequence A029790

- None of the digits in n is present in n^2 and n^3.
  - Currently known terms in the sequence : 2, 3, 7, 8, 22, 47, 53, 77, 92, 157, 187, 188, ..., 929522, 1717177, 8888588 (27 terms)
- Example:
  - when n = 22,
    n^2 = 484
    n^3 = 10648
  - when n = 47,
    n^2 = 2209
    n^3 = 103823
  - when n = **2**3,
    n^2 = 5**2**9
    n^3 = 1**2**167

- If it exists, a(28) > 10^9. - Derek Orr, Jul 13 2014
- If it exists, a(28) > 2 * 10^9. - Pratik Koirala, Eugene Fiorini, Otis Tweneboah, Nathan Fox, Jul 13 2015



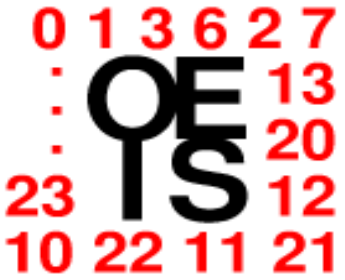
Sequence 248872

- Numbers n such that $n^n + n! + 1$ is prime.
  - Currently known terms: 1, 2, 4, 28, 496
- Example:
  - when n = 2:

    $2^2 + 2! + 1 =$ **7** ➔ Prime
  - when n = 4:

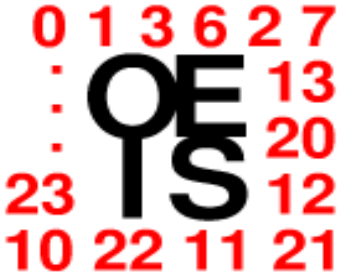
    $4^4 + 4! + 1 =$ **281** ➔ Prime
  - when n = 5:

    $5^5 + 5! + 1 =$ **3246** ➔ Not Prime
- If it exists, a(6) > 1500



Conclusion

- Analyzing the variation of A248872
- Analyzing the variation of A029790
- Search for other interesting sequences with the tags: more, hard, new and find new information about them
- Devise interesting variations of those sequences and publish them on the OEIS website



We would like to thank DIMACS, CCICADA, DHS, NSF.

Questions?