

Competition Graphs and Food Webs

Pratik Koirala

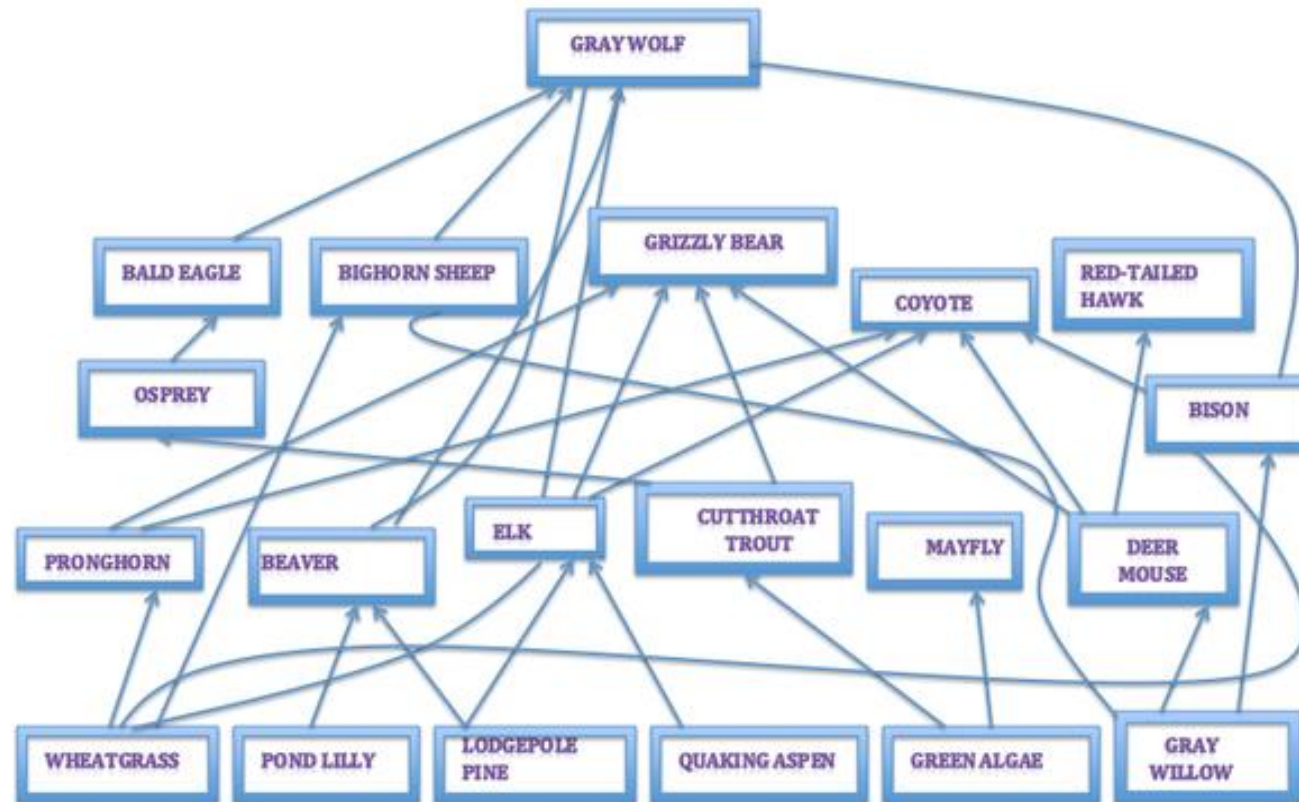
Mentor: Dr. Midge Cozzens

Overview

- Food Webs
- Competition Graphs
- Interval Graphs
- Research Problem
- Research Work
 - Transitive Orientation
 - Clique incidence matrix
 - Weighted model for food web
- Future Work
- Acknowledgement

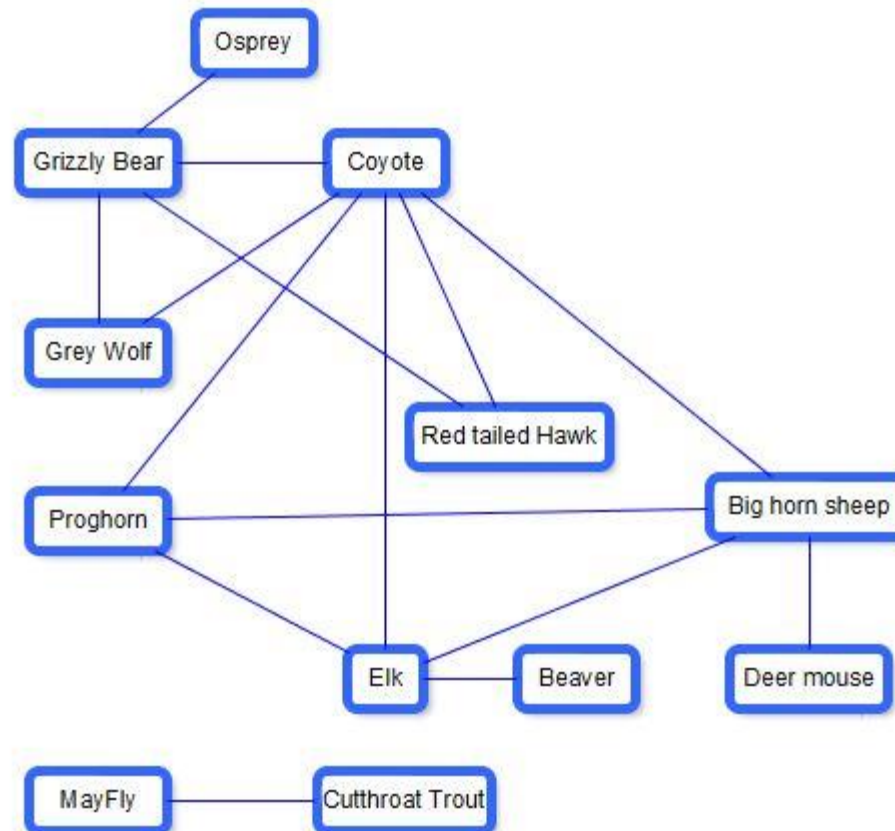
Food Webs

- Vertices in a directed graph (digraph) are species in an ecosystem, **assume no cycles**. Edges go from prey to predator.
- Example: Yellowstone National Park



Competition Graphs

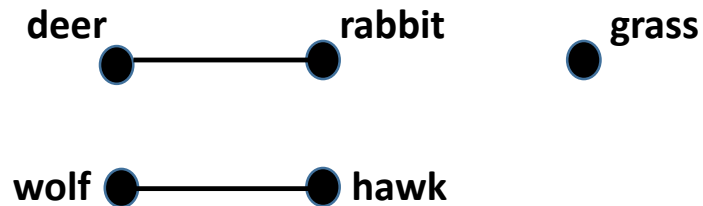
- For a digraph $D = (V, E)$, its corresponding competition graph: G is an undirected graph with vertex set V and there is an edge between two vertices if and only if they share a common prey.
- Example:



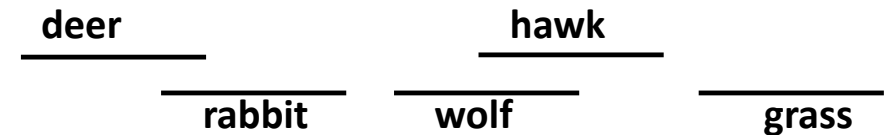
Interval Graphs

- Each vertex can be assigned an interval on the real line.
- There is an edge between two vertices if and only if their corresponding intervals overlap.

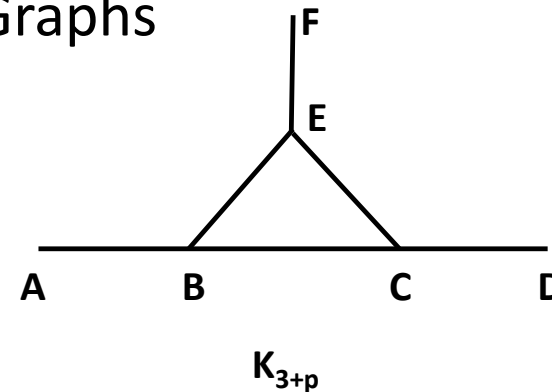
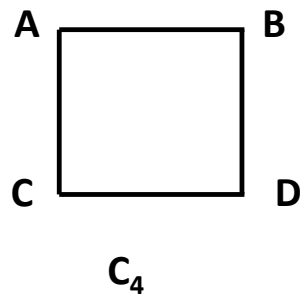
Graph G



Intervals corresponding to Graph G



- Some forbidden Subgraphs of Interval Graphs

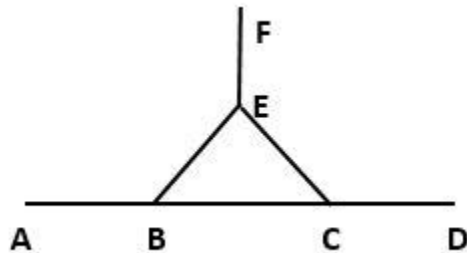


Research Problem

- For a competition graph to be an interval graph, both it and its complement must be chordal (no cycles of size 4 or more). Why?
- Can we characterize the directed graphs whose corresponding competition graphs are interval graphs?
- Most directed graphs do not have competition graph which are interval graphs, yet statistically most actual food webs have interval competition graphs. What ecological characteristics of food webs lead to this?

Research Work

- Transitive Orientation of the competition graph and its complement
- **Definition:** An undirected graph G has a transitive orientation if its edges can be oriented in such a way that, there is an edge (x, z) in the resulting directed graph if edges (x, y) and (y, z) exist in the resulting directed graph.
- Example:



➔ *No transitive orientation*

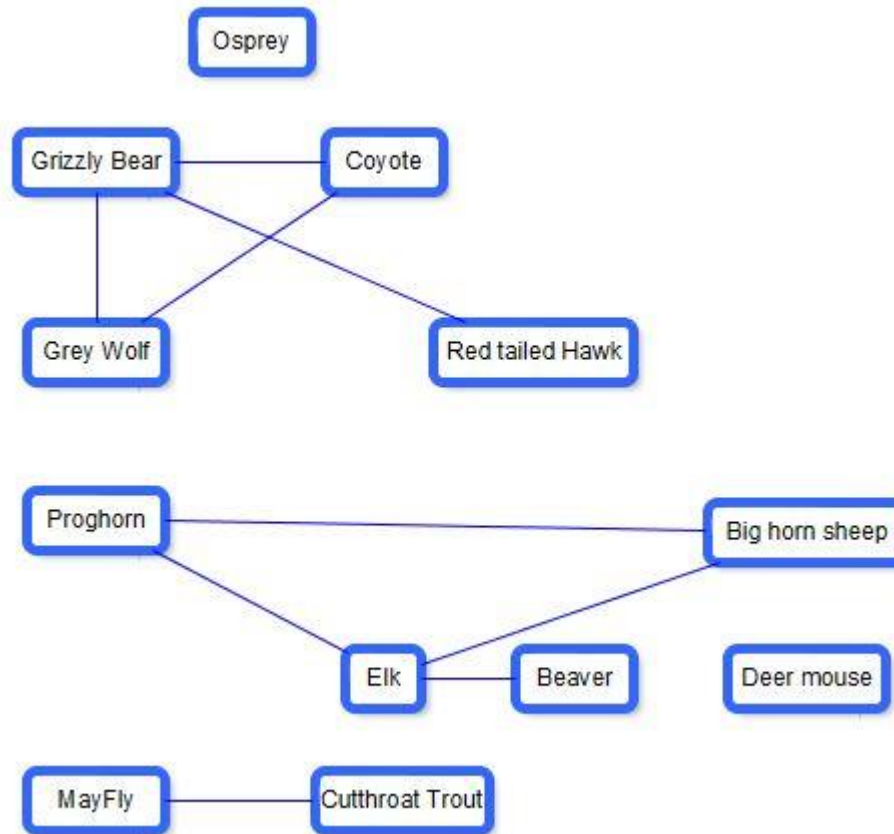
Consecutive Ones Property

- Consecutive Ones Property of the clique incidence matrix
 - A clique (maximal) is a complete subgraph
 - The rows of the matrix can be organized to get consecutive 1s in each column
 - For competition graphs of real food webs the maximal cliques have few vertices.
 - Example: Clique vs vertices binary matrix (for previous competition graph)

[illegible]

Weighted model for food web

- A weight of an edge represents the percentage of predator's diet
- Two species have edge in the competition graph if their edges from the common prey have weights greater than a threshold value
- Example: Threshold = 0.3



Future Work

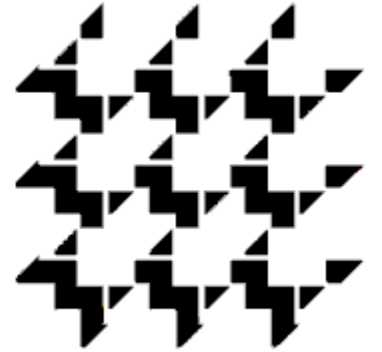
- Choosing an appropriate function to assign threshold
- Further explore the weighted model of food web

Acknowledgement



DIMACS

Center for Discrete Mathematics & Theoretical Computer Science
Founded as a National Science Foundation Science and
Technology Center



Thank you!