Generating functions (GF)

Basic GF

Let $a(x)$ be a GF and a_n is the corresponding sequence.

VF	**Explicit form of a sequence**
$a(x) = \frac{1}{1-x}$	$a_n = 1$
$a(x) = e^x$	$a_n = \frac{1}{n!}$
$a(x) = (1+x)^r$	$a_n = \binom{r}{n}$, where $\binom{r}{0} = 1$ a $\binom{r}{n} = \frac{r(r-1)\dots(r-n+1)}{n!}$

Operations with GF

Let $k \in \mathbb{N}$, $\alpha \in \mathbb{R}$, $a(x)$ and $b(x)$ are GFs for sequences a_n and b_n. We use operations to create $c(x)$ a GF with the corresponding sequence c_n.

Operation with GFs	**Formula of sequence**	
$c(x) := a(x) + b(x)$	$c_n = a_n + b_n$	
$c(x) := \alpha a(x)$	$c_n = \alpha a_n$	
$c(x) := a(\alpha x)$	$c_n = \alpha^n a_n$	
$c(x) := a(x^k)$	$c_{kn} = a_n$	$c_{n \not\equiv 0 \bmod k} = 0$
$c(x) := x^k a(x)$	$c_{n \neq \{0,\dots,k-1\}} = a_{n-k}$	$c_{\{0,\dots,k-1\}} = 0$
$c(x) := \frac{a(x) - \sum_{i=0}^{k-1} a_i x^i}{x^k}$	$c_n = a_{n+k}$	
$c(x) := a(x)b(x)$	$c_n = \sum_{i=0}^{n} a_i b_{n-i}$	
$c(x) := a'(x)$	$c_n = (n+1)a_{n+1}$	
$c(x) := \int a(x)\mathrm{d}x$	$c_{n\neq 0} = \frac{a_{n-1}}{n}$	$c_0 = 0$
$c(x) := \frac{a(x)}{1-x}$	$c_n = \sum_{i=0}^{n} a_i$	
$c(x) := a(x)^{-1}$, where $a_0 \neq 0$	$c_{n\neq 0} = \frac{\sum_{i=1}^{n} a_i c_{n-i}}{a_0}$	$c_0 = \frac{1}{a_0}$