REU Problems of the Czech Group

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Complexity of Fair Deletion Problems

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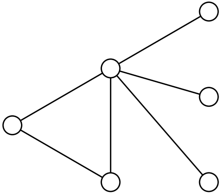

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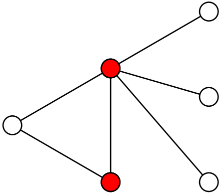

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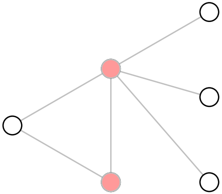

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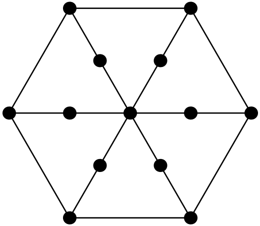
The goal is to find the smallest such set.

Fair Versions of Deletion Problems

Sometimes the smallest set is not what we want.

- We have a communication network, we want to remove redundancies
- We do not want to reduce the capability of each vertex by too much (a local condition in contrast to the previous global one)



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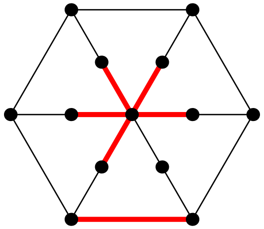

This solution removes a lot of links from single vertex.

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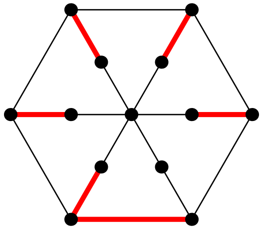

A much better solution.

Complexity of Graph Deletion Problems

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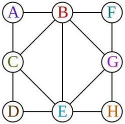

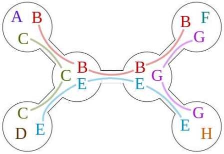

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For example, there is a parameter called tree-width that measures how much does the graph looks like a tree.

If we restrict ourselves to the graphs of bounded tree-width, we can get algorithms polynomial with respect to the size of a graph. (e.g. with running time $2^k n^3$, where k is the tree-width and n the size of a graph)

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- There is a result stating that every problem definable in a modification of MSO logic can be solved by an FPT algorithm (Kolman et al.)
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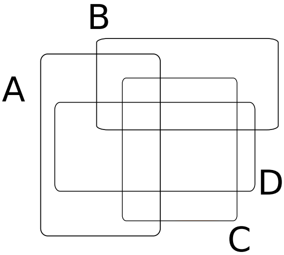
- The current parameterization is by tree-width; we want to explore different parameterizations that will lead to FPT algorithms

Simplification of the Inclusion-Exclusion Formula

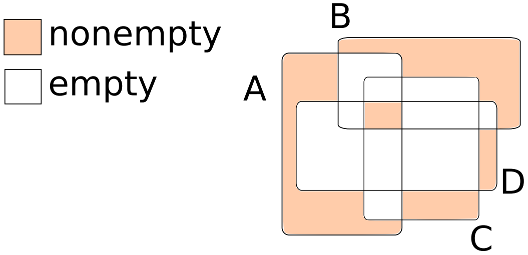
Inclusion-Exclusion Principle



$$
\begin{aligned}
|A \cup B \cup C \cup D| = \\
& |A| + |B| + |C| + |D| \\
& -|A \cap B| - |A \cap C| - |A \cap D| - |B \cap C| - |B \cap D| - |C \cap D| \\
& +|A \cap B \cap C| + |A \cap B \cap D| + |A \cap C \cap D| + |B \cap C \cap D| \\
& -|A \cap B \cap C \cap D|
\end{aligned}
$$

$$
\left|\bigcup_{i=1}^{n} A_i\right| = \sum_{\emptyset \neq J \subseteq \{1,2,\ldots,n\}} (-1)^{|J|-1} \left|\bigcap_{j \in J} A_j\right|
$$

Simplified Formula



$$|A \cup B \cup C \cup D| = |A| + |B| + |C| + |D| - 3|A \cap B \cap C \cap D|$$

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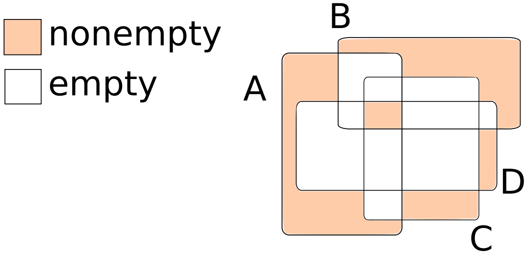

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Short formulas can be found by methods from linear algebra.

Simplified Formula 2

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Problem: In practice, evaluating formulas with large coefficients amplify measurement errors.

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We want the formula

- to be short in the number of terms,
- to have small coefficients,
- to be polynomially derivable.

Research Problems

Considering formulas with coefficients from the set $\{-1, 1\}$.

Q: Given m nonempty regions what is the shortest formula?

Q: Is it NP-hard to decide whether there is a formula with at most k terms?

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Q: Given m nonempty regions what is the shortest formula?

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Known results (Tancer et al.):

Lower Bound: There are families of sets requiring $\Omega(m^{2-\varepsilon})$ terms.

Upper Bound: There is an algorithm which finds a formula of size $m^{\mathcal{O}(\log^2(n))}$ in expected running time $m^{\mathcal{O}(\log^2(n))}$.

Stronger results for specific families of sets (e.g. disks in a plane).

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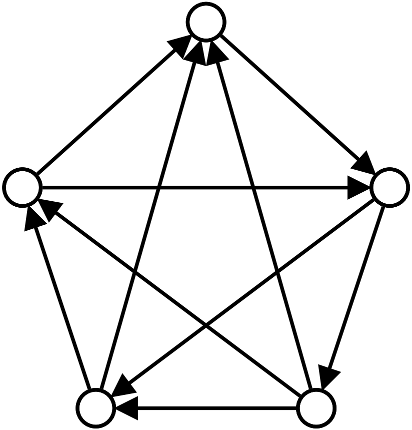
- Generalization of the results for specific families of set systems.
- Improving the upper bound.

Dominating Sets on Colored Tournaments

Dominating Set on Tournaments

- A tournament is a complete graph with orientated edges.
- A vertex v is dominated by u if there is an edge $u \to v$.
- A dominating set is a set of vertices called dominators such that every vertex of the graph is a dominator or is dominated by at least one.



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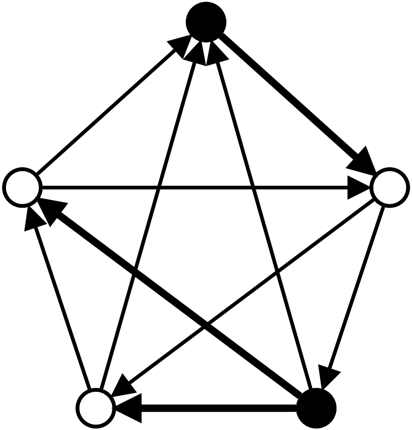
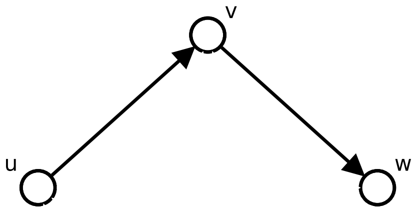


Edge-transitive Tournaments

- An edge-transitive digraph: $u \to v \& v \to w \implies u \to w$
- This means the graph must be acyclic.
- On transitive tournaments, one vertex dominates all.



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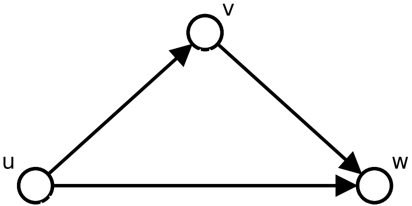

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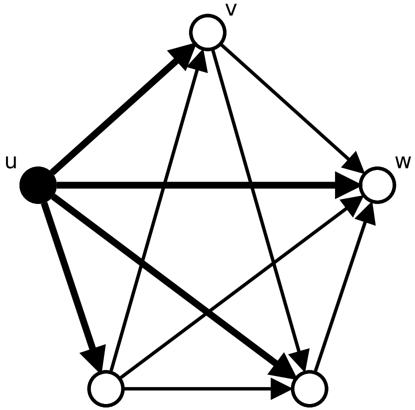

2-colored Transitive Tournaments

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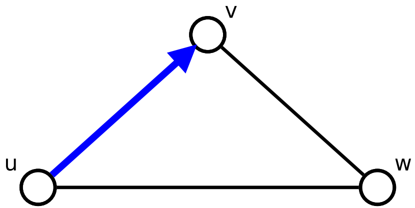

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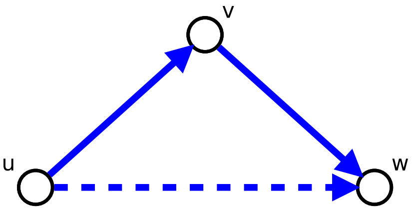

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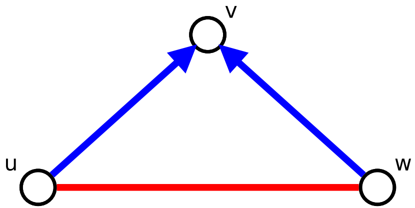

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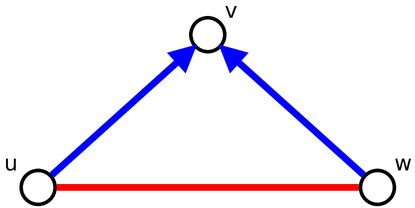
- Edges of the same color satisfy transitivity.
- Then the whole graph is transitive and it has a single dominator.



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Plan

At first we will test small cases algorithmically, try various tournament reductions and then generalize the results for tournaments of an arbitrary size.

Subclasses of Chordal Graphs

Symmetries of Graphs

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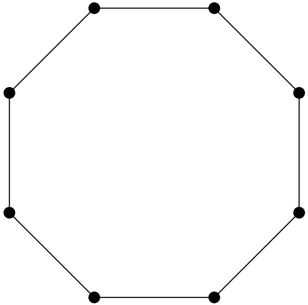

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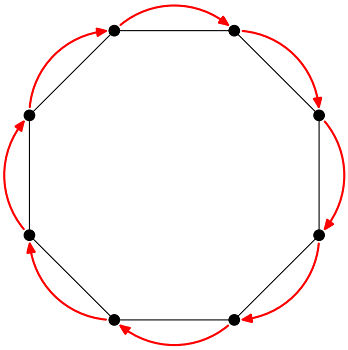

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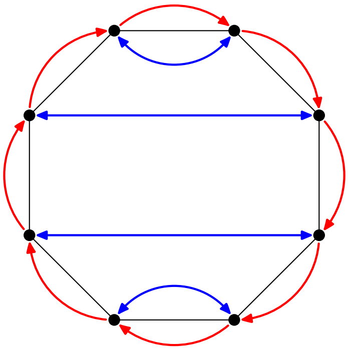
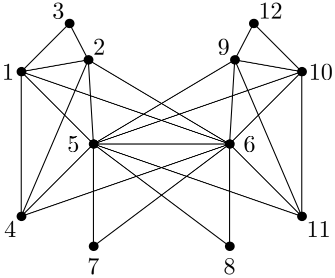
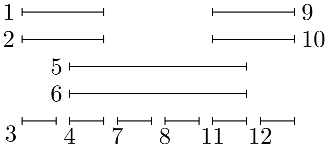


Interval Graphs (INT)

Interval graphs are intersection graphs of intervals of the real line.





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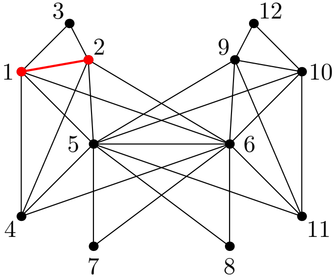

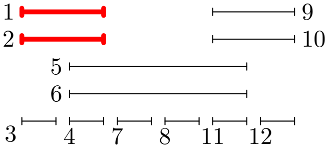

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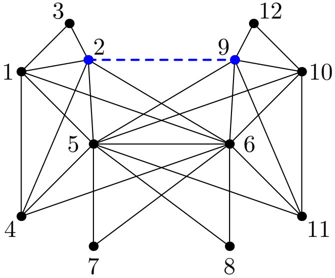

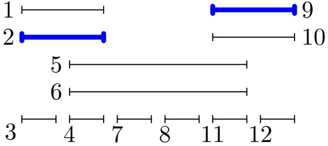

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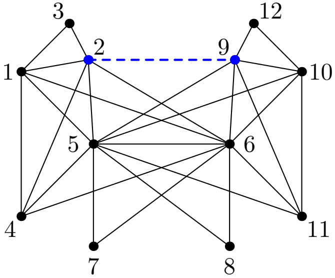

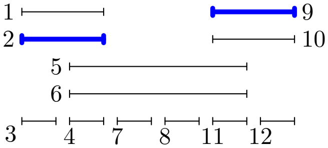

Many computational problems are polynomially solvable for interval graphs (graph isomorphism, maximum clique, k-coloring, maximum independent set, ...).

Chordal Graphs (CHOR)

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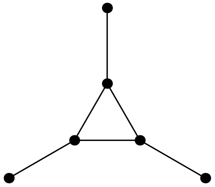

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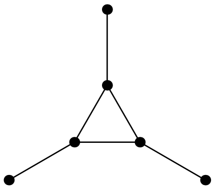

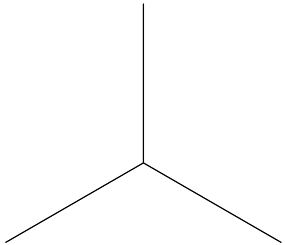

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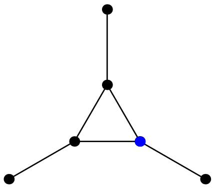

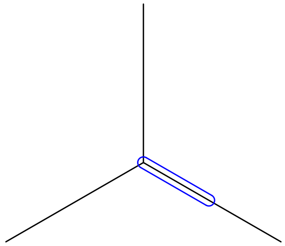

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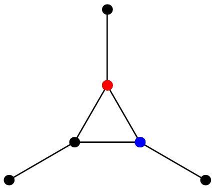
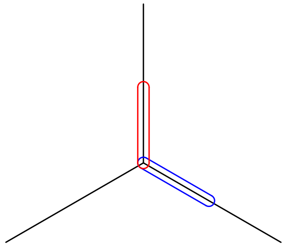
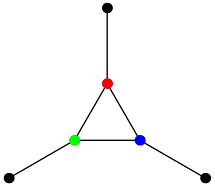
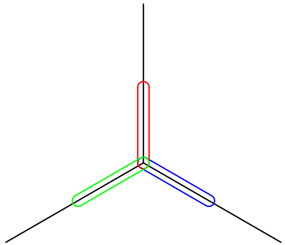




Chordal Graphs (CHOR)

Chordal graphs are intersection graphs of subtrees of a tree.





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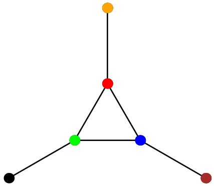

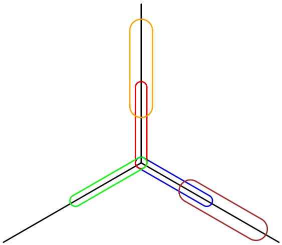

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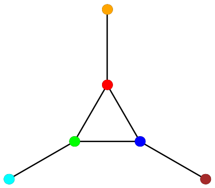

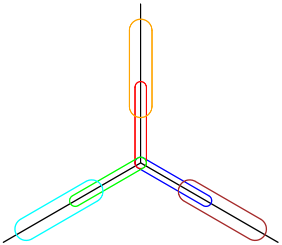

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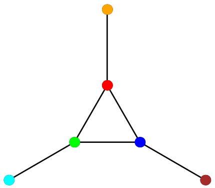

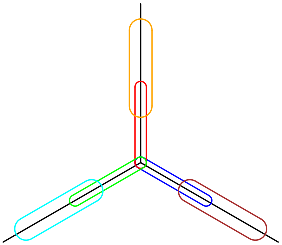

It is easy to see that INT $\subsetneq$ CHOR.

T-GRAPH

For a fixed tree T, the class of all intersection graphs of subtrees of T will be denoted by T-GRAPH.

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Concerning the class T-GRAPH, not much is known in comparison to INT and CHOR.

Main Aim

Our main aim is to study the structural and computational properties of the classes T-GRAPH.

First, we will try to understand the classes T-GRAPH, for some special tree T (for example $K_{1,n}$).

It is possible that some techniques developed for interval graphs could be used.

Thank you!