The Pinning Number of Overlapping Rectangles

Josef Cibulka
Jan Hladký
Alexandr Kazda
Bernard Lidický
Eva Ondráčková
Martin Tancer
and Vít Jelínek

DIMACS REU

July 19, 2006

The Problem

Let us have a set of axis parallel rectangles in the plane.

The Problem

Let us have a set of axis parallel rectangles in the plane.

We know that there are at most α mutually disjoint rectangles (the independence number is at most α).

The Problem

Let us have a set of axis parallel rectangles in the plane.

We know that there are at most α mutually disjoint rectangles (the independence number is at most α).

We want to pin them down so that each rectangle is pinned at least once.

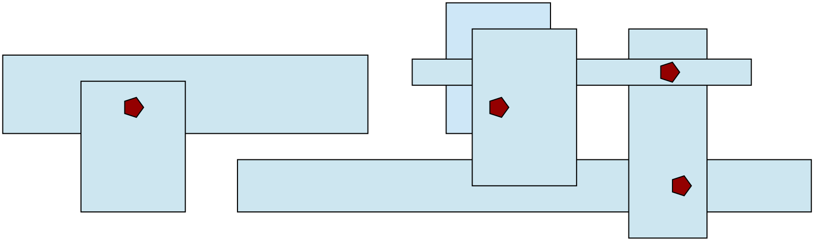
The Problem

Let us have a set of axis parallel rectangles in the plane.

We know that there are at most α mutually disjoint rectangles (the independence number is at most α).

We want to pin them down so that each rectangle is pinned at least once.

How many pins do we need?



The Project

It is known that $O(\alpha \cdot \log_2 \alpha)$ pins suffice.

The Project

It is known that $O(\alpha \cdot \log_2 \alpha)$ pins suffice.

G. Wegner conjectured in 1965 that we always need at most $2\alpha - 1$ pins.

The Project

It is known that $O(\alpha \cdot \log_2 \alpha)$ pins suffice.

G. Wegner conjectured in 1965 that we always need at most $2\alpha - 1$ pins.

During our project, we have tried to decide, whether $O(\alpha)$ pins are enough.

The Project

It is known that $O(\alpha \cdot \log_2 \alpha)$ pins suffice.

G. Wegner conjectured in 1965 that we always need at most $2\alpha - 1$ pins.

During our project, we have tried to decide, whether $O(\alpha)$ pins are enough.

Unfortunately, the general case has proven to be too hard to answer.

The Project

It is known that $O(\alpha \cdot \log_2 \alpha)$ pins suffice.

G. Wegner conjectured in 1965 that we always need at most $2\alpha - 1$ pins.

During our project, we have tried to decide, whether $O(\alpha)$ pins are enough.

Unfortunately, the general case has proven to be too hard to answer.

However...

Our Results

We know that:

- If there are no crossings then $O(\alpha)$ pins suffice.

Our Results

We know that:

- If there are no crossings then $O(\alpha)$ pins suffice.
- It is enough to consider the case of rectangles with independence α on $\alpha \times \alpha$ grid.

Our Results

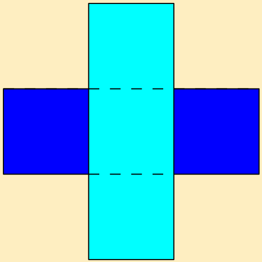
We know that:

- If there are no crossings then $O(\alpha)$ pins suffice.
- It is enough to consider the case of rectangles with independence α on $\alpha \times \alpha$ grid.
- If $k\alpha$ pins suffice, then $k \geq 2$.

Crossings are Bad

Theorem

There is k such that $k\alpha$ pins suffice to pin down any set of rectangles that does not contain two that intersect in this way

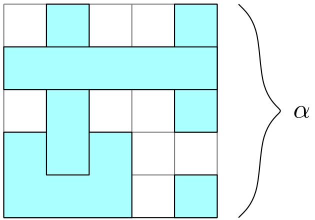


We will omit the proof which is a bit technical.

Reduction to $\alpha \times \alpha$ Grid

Theorem

If we can pin down any set of independence α on $\alpha \times \alpha$ grid (see figure) with $k\alpha$ pins, then we can pin down any set of independence α by at most $9k\alpha$ pins.



Asymptotics

Theorem

If $k\alpha$ pins always suffice to pin down any set of independence α, then $k \geq 2$.

Asymptotics

Theorem

If $k\alpha$ pins always suffice to pin down any set of independence α, then $k \geq 2$.

Asymptotics

Theorem

If $k\alpha$ pins always suffice to pin down any set of independence α, then $k \geq 2$.

Sketch of a proof: For any given n we will construct a set of independence at most $n + 2$ for which we need at least $2n$ pins.

Asymptotics

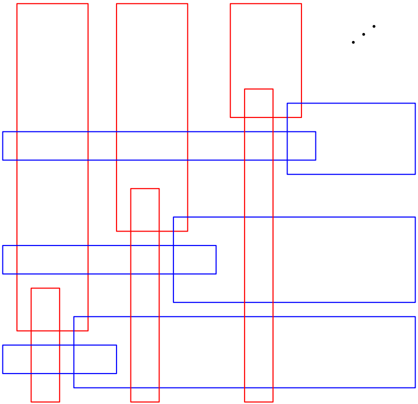
Theorem

If $k\alpha$ pins always suffice to pin down any set of independence α, then $k \geq 2$.

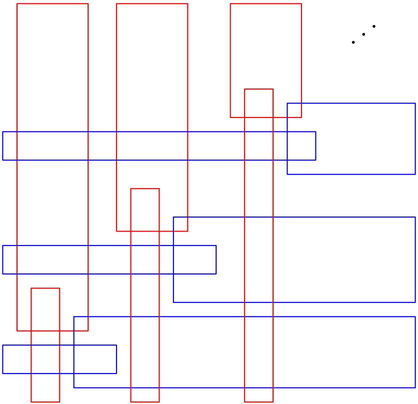
Sketch of a proof: For any given n we will construct a set of independence at most $n+2$ for which we need at least $2n$ pins.

For n big it is $\frac{2n}{n+2} \to 2$.

Sketch of a Proof, continued

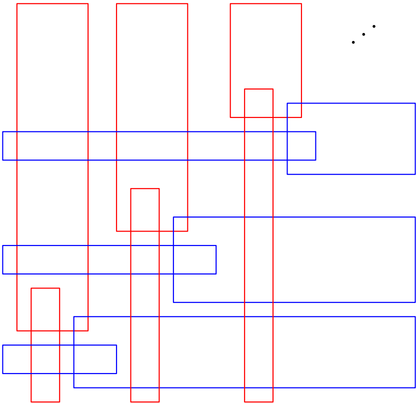


Sketch of a Proof, continued



Since there are $4n$ rectangles and each point lies in at most 2 rectangles we need at least $2n$ pins.

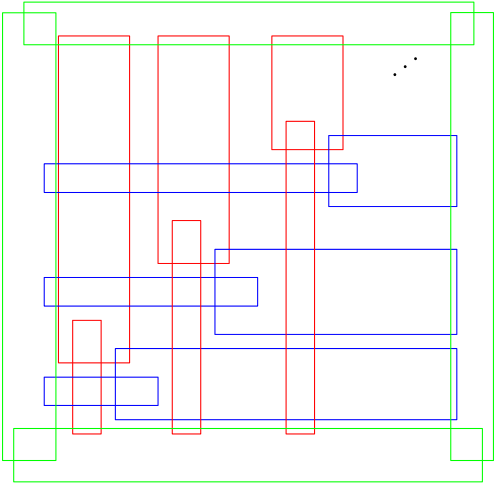
Sketch of a Proof, continued



Since there are $4n$ rectangles and each point lies in at most 2 rectangles we need at least $2n$ pins.

It can be shown that no more than $n + 2$ rectangles from this set can be mutually disjoint.

An Improvement



It can be shown that this set has (for $n \geq 3$) independence number $n+2$ and we need at least $2n+2$ pins.

Conclusions

While we have not managed to solve the general problem, we have at least simplified it.

Conclusions

While we have not managed to solve the general problem, we have at least simplified it.

The fact that for many configurations we need only linearly many pins hints that the number of pins needed is indeed linear in α.

Thanks

We would like to thank our mentor Mario Szegedy.

We would also like to thank János Komlós for his valuable advice and József Beck for suggesting the problem.

Thanks

We would like to thank our mentor Mario Szegedy.

We would also like to thank János Komlós for his valuable advice and József Beck for suggesting the problem.

Thank you for your attention.