

$$\begin{aligned}
H(n, i, j) = \binom{4}{j} \binom{4}{i} & \left[\binom{12}{n+i+j-8} - x_{n+i+j-8} \right] - \left[\binom{4}{j} \binom{3}{i} + \binom{2}{j-1} + \binom{1}{j-1} + \binom{0}{j-1} \right] \binom{6}{n+i+j-13} \\
& - \left[\binom{4}{j} \binom{2}{i} + \binom{1}{j-1} + \binom{0}{j-1} \right] \binom{7}{n+i+j-12} \\
& - \left[\binom{4}{j} \binom{1}{i} + \binom{0}{j-1} \right] \binom{8}{n+i+j-11} \\
& - \left[\binom{4}{j} \binom{0}{i} \right] \binom{9}{n+i+j-10}
\end{aligned}$$

So

$$\begin{aligned}
H(n, i, j) &= \binom{4}{j} \binom{4}{i} \left[\binom{12}{n+i+j-8} - x_{n+i+j-8} \right] \\
&- \sum_{k=0}^3 \left(\binom{4}{j} \binom{k}{i} + \binom{k-1}{j-1} + \binom{k-2}{j-1} + \binom{k-3}{j-1} \right) \binom{9-k}{n+i+j-k-10}
\end{aligned}$$

The quantity we are interested in is

$$2^{20} \sum_{j \leq 4} \sum_{0 \leq i < j} \sum_{n=6}^{11} H(n, i, j)$$

.