Online Matching for Internet Auctions

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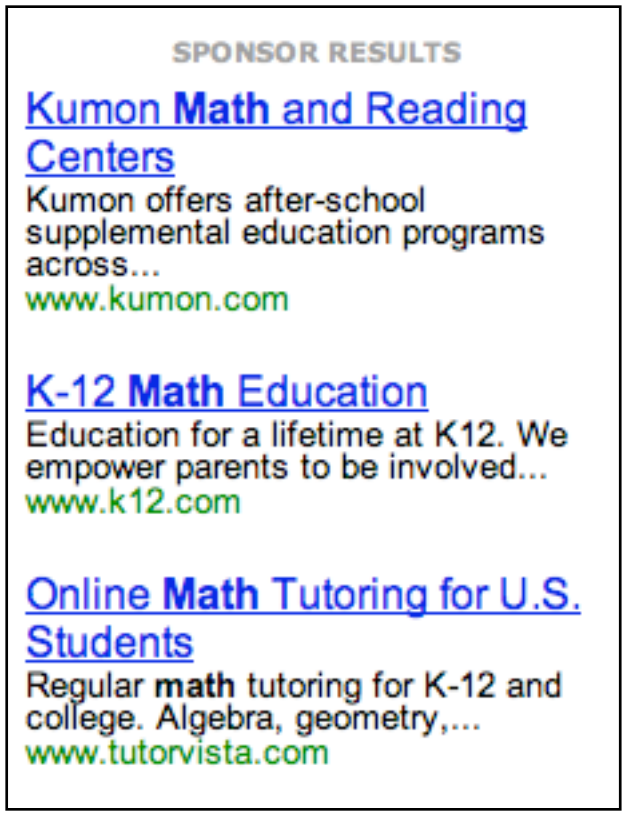
Internet Auctions

- Every time a search query is entered, an automated auction is run to determine which ads are displayed.
- Each advertiser specifies a daily budget and some number of bids.
- The goal is to maximize revenue without exceeding any advertiser's budget.



Ads from http://www.yahoo.com

Online Bipartite Graph Matching

Online Problem

- Given one query at a time, can we decide which advertiser to match it to?
- Evaluated using the competitive ratio, which is the worst case ratio between the online algorithm and the optimal solution.

Offline Problem

- Given all of the queries, can we find a matching that optimizes revenue?
- This problem NP-hard. The best we can do in general is $O(n4^k)$ where *n* is the number of bidders and *k* is the number of queries.

(Andelman and Mansour, 2004)

The Algorithm

- Mehta et. al. describe an online algorithm that gives a competitive ratio of $1 - 1/e$
- They define the tradeoff function:

$$\psi(x) = 1 - e^{-(1-x)}$$

- Assign each query to the bidder i that maximizes $b_i\psi(T(i))$ where b_i is i's bid, and $T(i)$ is the fraction of i's budget spent so far.
- Easily modifiable to incorporate a variety of auction techniques.

What's Next

- Use LP rounding to approximate the solution to the off-line problem and estimate the competitive ratio.
- Consider special cases (i.e. when each advertiser bids on many queries) and see if we can do better.
- Develop effective heuristics, even if we can't prove the results.

References

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