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Abstracts of Talks

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Flows, View-Obstructions and the Lonely runner

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We prove the following result.

Theorem. *Let G be an undirected graph. If G has a nowhere zero flow with at most k different values, then it also has one with values from the set $\{1, \dots, k\}$.*

When $k \geq 5$, this is a trivial consequence of Seymour's "six-flow theorem". When $k \leq 4$ our proof is based on a lovely number theoretic problem which we call the "Lonely Runner Conjecture".

Conjecture. *Suppose k runners having nonzero constant speeds run laps on a unit-length circular track. Then there is a time at which all runners are at least $1/(k+1)$ from their common starting points.*

This conjecture appears to have been formulated by J. Wills and independently by T. Cusick. Fortunately for our purposes, this conjecture has been verified for $k \leq 4$ by Cusick and Pomerance in a complicated argument involving exponential sums and electronic case checking. A major part of this paper is an elemenatery self-contained proof of the case $k = 4$ of the Lonely Runner Conjecture.

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List group coloring is Π_2^P-complete

D. Král', P. Nejedlý

A graph G is A-ℓ-choosable for an Abelian group A and an integer $\ell \leq |A|$ if for each orientation of G, each edge-labeling $\varphi : E(G) \to A$ and each list-assignment $L : V(G) \to \binom{A}{\ell}$, there exists a vertex-coloring $c : V(G) \to A$ with $c(v) \in L(v)$ for each vertex v and with $c(v) - c(u) \neq \varphi(uv)$ for each oriented edge uv of G. We prove a dichotomy result on the computational complexity of this problem. In particular, we show that the problem is Π_2^P-complete if $\ell \geq 3$

for any group A and it is polynomial-time solvable if $\ell = 1, 2$. This also settles the complexity of group coloring for all Abelian groups.

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Crown Reductions in graph problems

We give a short presentation of parameterized complexity, and then present Crown-decomposition as a tool for graph reductions. We motivate further studies on Crown-decomposition by a survey of graph-problems where the best known kernels are obtained from crown-decomposition type reductions.

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The Graph Reconstruction Problem

Let G be a graph on n vertices, and let $\mathcal{D}_G = \{G - v | v \in G\}$ be the family of maximal vertex-deleted subgraphs of G. In 1941 Ulam stated the following conjecture.

Reconstruction Conjecture. *Every graph G on at least three vertices can be reconstructed up to isomorphism from $\mathcal{D}_G$.*

To simplify the terminology, we imagine that we draw each of the subgraphs in $\mathcal{D}_G$ on a card, and we call $\mathcal{D}_G$ itself the **deck** of the graph G. If G can be reconstructed from $\mathcal{D}_G$, we say that G is **reconstructible**.

If G has two vertices, then the deck consists of two cards, each with one vertex and no edges; in this case we are not able to determine whether $G = K_2$ or $G = N_2$. (K_2 and N_2 are the complete graph and the empty graph on two vertices, respectively.) Apart from this example there are no graphs known to be not reconstructible.

Given the deck $\mathcal{D}_G$ of a graph, it is natural to start by determining the number of vertices and edges in G. The number of vertices can be found by observing that it is equal to the number of cards, or alternatively that it is one more than the number of vertices on any card in the deck. To find the number of edges, we observe that any edge in G is visible on all but two of the cards in the deck. Hence,

$$|E(G)| = \sum_{G-v \in \mathcal{D}_G} \frac{|E(G-v)|}{|V(G)|-2}.$$

For any card $G - v$ we can then find the degree of v, since this equals $|E(G)| - |E(G-v)|$.

This can be generalized to any subraph of G with fewer vertices than G; the following lemma was first published by Kelly in [3].

Lemma 1. *Let F be a graph with fewer vertices than G. Then it is possible to determine from $\mathcal{D}_G$ the number of subgraphs of G that are isomorphic to F. Furthermore, for any card $G - v$ in $\mathcal{D}_G$, we can determine the number of subgraphs of G that are isomorphic to F and which contain v.*

Proof. Let $\sigma_G(F)$ be the number of subgraphs of G that are isomorphic to F. Then

$$\sigma_G(F) = \sum_{G-v \in \mathcal{D}_G} \frac{\sigma_{G-v}(F)}{|V(G)| - |V(F)|}.$$

Since every term on the right hand side in the equality can be determined, we can calculate $\sigma_G(F)$.

Given a card $G - v$, the number of subgraphs of G which are isomorphic to F and contain v, is $\sigma_G(F) - \sigma_{G-v}(F)$. q.e.d.

If $\mathcal{G}$ is a class of graphs, we say that $\mathcal{G}$ is **recognizable** if we for any deck $\mathcal{D}_G$ can determine whether $G \in \mathcal{G}$ or $G \notin \mathcal{G}$.

Let $\mathcal{B}$ be the class of bipartite graphs. It is easy to see that $\mathcal{B}$ is recognizable: If G is not bipartite, then at least one of the cards in $\mathcal{D}_G$ is also not bipartite, except for the case that G is an odd cycle. It follows that we can always determine whether G is bipartite or not.

Furthermore Salvi proved in [1] that one can determine the size and degree sequences of the colour classes.

It is proved in [2] that if G is a bipartite graph such that one of the colour classes contains at most five vertices, (and the other colour class contains an arbitrary number of vertices,) then G is reconstructible. [2] also contains some techniques which can be used to reconstruct bipartite graphs, some of which are outlined here.

Suppose that $\mathcal{D}_G$ is the deck of a bipartite graph. We want to split the deck into two subdecks, corresponding to the colour classes. That is, for any card $G - v$, we want to determine whether $v \in V_1$ or $v \in V_2$. We say that a card $G - v$ is **semiorientable** if this can be done. A card is **orientable** if we in addition are able to determine whether $w \in V_1$ or $w \in V_2$ for every vertex $w \in G - v$.

If the maximal degree in V_1 differs from the maximal degree in V_2, then all the cards in $\mathcal{D}_G$ are semiorientable. This follows from the following lemma.

Lemma. *For any card $G - v$ in $\mathcal{D}_G$ we can determine the distance in G between v and the closest vertex of degree $\Delta(G)$, where $\Delta(G)$ is the maximal degree in G.*

Proof. A bomb B is a tree, with a vertex c, such that c is the only vertex of degree greater than two, and such that c has only one neighbour of degree greater than one. Let z be the vertex in B furthest from c. The path from c to z is the fuse of the bomb. A bomb is determined by the degree of c, and the length of the fuse, and we let B_l^d be the bomb such that c has degree d and the fuse has length l.

Given a card G_v, we can, according to Lemma 1, always determine whether G contains a subgraph containing v which is isomorphic to $B_l^{\Delta(G)}$. Let l be the smallest l such that v is contained in such a subgraph. Then l is the distance between v and the closest vertex of degree $\Delta(G)$. q.e.d.

As mentioned above, we can determine the degree sequence of V_1 and V_2. So if the maximal degree in V_1 is different from the maximal degree in V_2, then for any card $G - v$, we can determine whether $v \in V_1$ or $v \in V_2$ by considering the parity of the distance between v and a vertex of degree $\Delta(G)$.

The following two results are proved in [2].

A bipartite graph is k-**expanding** if, for any set of vertices $W \subseteq V_1$, we have $|N(W)| \geq |W| + k$.

Lemma. *If G is a 1-expanding bipartite graph, all the cards in the deck of G are semiorientable. If G is 2-expanding, then all the cards are orientable.*

Theorem. *Let G be a bipartite graph such that $||V_1| - |V_2|| > 1$ and such that V_2 contains a vertex which is adjacent to all the vertices in V_1. Then G is reconstructible.*

Theorem. *If G is a bipartite graph with $|V_1| \leq 5$, then G is reconstructible. If G is a bipartite graph with $|V_1| = 6$, then all the cards in $\mathcal{D}_G$ are orientable.*

[1] N. M. Salvi: *Reconstructing some invariants of a bipartite graph,* J. Comb. Inf. Syst. Sci., 1983, vol. 8, 5–9

[2] T. G. Seierstad: *Ulam's reconstruction problem for graphs,* University of Bergen, 2002

[3] P. J. Kelly: *A congruence theorem for trees,* Pac. J. Math., 1957, vol. 7, 961–968

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The chromatic number of the product of two graphs is at least half the minimum of the fractional chromatic numbers of the factors

Claude Tardiff

One outstanding problem in graph theory is a formula concerning the chromatic number of the product of two graphs. In 1966, Hedetniemi conjectured that

$$\chi(G \times H) = \min\{\chi(G), \chi(H)\},$$

and a proof of this formula has not yet been found.

The article presents a result concerning an alternative notion of a chromatic number, the so called fractional chromatic number. Claude Tardiff shows that for the fractional chromatic number χ_f the following inequality holds:

$$\chi(G \times H) \geq \min\{\chi_f(G), \chi_f(H)\}.$$

According to the author, this might suggest the function

$$f(n) = \min\{\chi(G \times H) : \chi(G) \geq n, \chi(H) \geq n\}$$

concerning the usual chromatic number is unbounded, which has not yet been decided.

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On a theorem of Jordan

Jean-Pierre Serre

The talk was based on the paper "On a theorem of Jordan" by Jean-Pierre Serre. After a brief reminder of some definitions and facts in group theory, I presented Jordan's theorem stating that in a transitive group acting on more than one point there always is an alement with no fixed points (a derangement).

This theorem was first presented by C. Jordan in 1872. Moreover, it was proved by Cameron and Cohen in 1992, that the proportion of derangements is at least $1/n$ where n is the number of points the group acts on.

As a consequence, I presented the main result of the cited paper. If f is a polynomial of degree n with integer coefficients, let $N_p(f)$ be the number of zeroes of f over $Z_p = Z/pZ$. If $n \geq 2$ and f is irreducible over the rationals, then there are infinitely many primes such that $N_p(f) = 0$; moreover, such primes have a density (in the set of all primes) that is at least $1/n$.

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An algorithm for source location in directed graphs

Mihály Bárász, Johanna Becker, András Frank

We provide a strongly polynomial-time algorithm for the following problem. Given a directed graph D and nonnegative integers k and l, find a minimum-cardinality subset R of nodes such that for any node v of D not in R, there exist k pairwise edge-disjoint directed paths from R to v, and l pairwise edge-disjoint directed paths form v to R. A subset of nodes satisfying these connectivity properties is called a (k, l)-*source*. We call a subset of nodes deficient if it's in-degree is less than k or it's out-degree is less than l. The motivation for solving this problem comes from the paper IMAIF by Ito, Makino, Arata, Honami, Itatsu, and Fujishige, who showed that *(i)* the problem can be solved in polynomial time for fixed k and l, and *(ii)* there is a min-max theorem for this problem:

Theorem ([1]). *The minimum cardinality of a (k, l)-source is equal to the maximum number of pairwise disjoint deficient sets.*

They proved this result by observing first, that in order to cover all deficient sets, it is sufficient to cover only the inclusionwise minimal ones. Second, they proved that the family of all minimal deficient sets form a subtree hypergraph H_{kl}. Finally, they invoked the known result that in any subtree hypergraph H the transversal number $\tau(H)$ is equal to the matching number $\nu(H)$.

A *subtree hypergraph* (aka. hypertree or arboreal hypergraph) is one for which there is a tree T on its node set so that each hyperedge induces a subtree of T. The tree is called a *basic* (or representative) *tree* for the hypergraph. By a known characterization a hypergraph is a hypertree iff its line graph is chordal and it admits the Helly property (intersection of a pairwise intersecting subset of its edges is nonempty). And there is an algorithm, due to F. Gavril [2], to

compute a maximum stable set in a chordal graph along with a minimum set of cliques covering all nodes. This provides an algorithm for our problem, since a stable set of nodes in $L(H_{kl})$ corresponds to a set of pairwise disjoint deficient sets, while a clique covering of $L(H_{kl})$ corresponds to a (k, l)-source.

Unfortunately this algorithm is not polynomial if k, l are not fixed, as H_{kl} can be exponentially large in size of D (one can easily construct an example for this). Our approach gets around the exponentiality of H_{kl} as follows. First, we show that it is sufficient to work with a tree representation T of H_{kl}, rather then H_{kl} itself. That is, a maximum matching of H_{kl} can be computed from directly T and D. The only subroutine we need for this it to decide whether a node set contains a minimal deficient set, which can be decided by running two simple flow algorithms.

The main difficulty in this approach is in the computation of T. We circumvent this difficulty in two steps. First, seemingly a contradiction, we introduce a larger hypergraph which is still a hypertree. But then we show that there is a "surrogate" of it, a subhypergraph which is sufficient to compute T and is polynomial in size of D .

Definition. *Given a digraph $D = (V, A)$, we call a nonempty subset Z of V in-solid (respectively, out-solid) if $\varrho(X) > \varrho(Z)$ (respectively, $\delta(X) > \delta(Z)$) for every nonempty proper subset X of Z. An in- or out-solid set is called solid. Singletons are always in- and out-solid, and a minimal k-in-deficient set is in-solid (for any k). Let H_D denote the hypergraph of all solid sets.*

An other way for looking at the hypergraph H_D is that it is the union of hypergraphs H_{kl} for all possible k and l. The usefulness of the notion of solid sets lies in the following property.

Lemma. *Union of two intersecting solid sets is solid.*

By a *maximal s-avoiding solid* set we mean a solid set which is a subset of $V - s$ not included in any other solid subset of $V - s$. By the above lemma it follows that the distinct maximal s-avoiding solid sets are disjoint, that is the family of maximal s-avoiding solid sets is a partition of $V - s$. Let H'_D denote the subhypergraph of H_D consisting of those hyperedges which are maximal s-avoiding solid sets for some $s \in V$. Note that H'_D has at most n^2 hyperedges, that is, H'_D is small even if H_D has exponentially many hyperedges. Therefore a basic tree for H'_D is computable in polynomial time. The good thing is that this tree is automatically basic for the whole H_D. This is simply because every hyperedge of H_D is an intersection of some hyperedges in H'_D.

This finishes the description of our approach. The algorithm we designed runs in $O(n^3 m \log(n^2/m))$ (for this an algorithm by Hao and Orlin [3] is used to speed up the computation of solid partitions) and can be used without modifications for the capacitated version of the problem.

[1] Hiro Ito, Kazuhisa Makino, Kouji Arata, Shoji Honami, Yuichiro Itatsu, and Satoru Fujishige, Source location problem with flow requirements in directed networks, Optimization Methods and Software, Vol. 18, No. 4, August 2003, pp. 427-435.

[2] F. Gavril, Algorithms for minimum coloring, maximum clique, minimum covering by cliques, and maximum independent set of a chordal graph, SIAM Journal on Computing 1 (1972) 180-187.

[3] J. Hao and J. B. Orlin, A faster algorithm for finding the minimum cut in a graph, J. Algorithms 17 (1994) 424-446.

[4] Jan van den Heuvel and M. Johnson, A polynomial algorithm for the source location problem in digraphs, CDAM Research Report, LSE-CDAM-2004-02.

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Recognizing P_3-structure: A Switching Approach

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Polynomial time recognition of P_4-structure

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Let G be a fixed unoriented graph. The G-structure of a graph F is the hypergraph H with the same set of vertices as F and with the property that a set h is a hyperedge of H iff the subgraph of F induced on h is isomorphic to G. We consider the complexity of determining whether for a given hypergraph H there exists a graph F such that H is the G-structure of F.

The presented papers prove that the problem is polynomial for $G = P_3$ and $G = P_4$.

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Faster shortest-path algorithms for planar graphs

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We give a linear-time algorithm for single-source shortest paths in planar graphs with nonnegative edge lengths. The previous best algorithms for these problems required $\Omega(n\sqrt{\log n})$ time where n is the number of vertices in the input graph. We use graph decopositions based on separators.

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Identities of finite groups

Let $\mathcal{A}$ be an algebra. Let us denote the difficulty of determining whether two terms are the same to all substitutions in $\mathcal{A}$ with Term-EQ $(\mathcal{A})$. Let us denote the same problem to polynomials (terms with constants) with Pol-EQ $(\mathcal{A})$. We also can check whether an equation can be satisfied in $\mathcal{A}$ or not, let us denote its difficulty with Pol-SAT $(\mathcal{A})$.

Burris – Lawrence and Szabó – Vértesi proved dihotomy for rings, the problems here are fully solved. For semigroups we almost know nothing, the few known results belong to McKenzie, Moore, Tesson, Thérien, Volkov, Szabó, Vértesi. In groups Burris – Lawrence proved that if G is a nilpotent group, then the Term-EQ (G) problem is in P, or if G is non-solvable then Term-EQ (G) is coNP-complete. Goldmann – Russel proved that solving system of equations is in P for abelian groups and coNP-complete otherwise. We could prove the following:

G. Horváth, Cs. Szabó. *If $G \simeq A \otimes B$ where A is abelian and* Term-EQ (B) *is in P then* Term-EQ (G) *is in P.*

This solves the problem e. g. for meta-abelian groups, S_4, A_4, D_n.

Let us denote with Term-EQ* and Pol-SAT* the former problems when we have the possibility to preprocess some new operations with the operations given by the algebra. Idziak – Szabó with TCT proved that in a congruence-modular variety Pol-SAT* $(\mathcal{A})$ is in P for nilpotent $\mathcal{A}$ algebras and NP-complete otherwise. This result does not say anything about terms. We proved that

G. Horváth, Cs. Szabó. *If G is a nilpotent group then* Pol-SAT* (G) *is in* P. *If G is not nilpotent then Term-EQ* (G) is coNP-complete.*

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Essential covers of the cube by hyperplanes

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A set L of linear polynomials in variables $X_1, X_2, \ldots, X_n$ with real coefficients is said to be an essential cover of the cube $\{0,1\}^n$ if (E1) For each $v \in \{0,1\}^n$, there is a $p \in L$ such that $p(v) = 0$; (E2) No proper subset of L satisfies E1, that is, for every $p \in L$, there is a $v \in \{0,1\}^n$ such that p alone takes the value 0 on v; (E3) Every variable appears (in some monomial with non-zero coeffcient) in some polynomial of L. Let $e(v)$ be the size of the smallest essential cover of $\{0,1\}^n$. In this paper it is showed that

$$\sqrt{n} + 1 \leq e(n) \leq \lceil \frac{n}{2} \rceil + 1.$$

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Error-Free and Best-Fit Extensions of Partially Defined Boolean Functions

Endre Boros, Toshihide Ibaraki, Kazuhisa Makino

In this paper, we address a fundamental problem related to the induction of Boolean logic: Given a set of data, represented as a set of binary "true n-vectors" (or "positive examples") and a set of "false n-vectors" (or "negative examples"), we have to establish a Boolean function (extension) f with some specified properties, so that f is true (resp. false) in every given true (resp. false) vector. We shall study this problem in the presence of some a priori knowledge or hypothesis about the extension f. Such knowledge may be obtained from experience or from the analysis of mechanisms that may or may not cause the phenomena under consideration. The real-world data may contain errors, e.g. measurement errors might come in when obtaining data, or there may be some other influential factors not represented as variables in the vectors. To cope with such situations, we may have to give up the goal of establishing an extension that is perfectly consistent with the given data. If there is no such extension, the best we can expect is to establish an extension f, which has the minimum number of misclassifications.

Both problems, i.e. the problem of finding an extension within a specified class of Boolean functions, and the problem of finding a minimum error extension in that class, will be extensively studied in this paper. For certain classes we shall provide polynomial algorithms, and for other cases we prove their NP-hardness.

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Packing Cycles in classes of perfect graphs

Jan Kára and Ton Kloks

We investigate the computational complexity of the question of finding the maximum number of vertex disjoint cycles in an input graph, restricted to some classes of perfect graphs. We show that the problem becomes tractable in polynomial time when restricted to splitgraphs or cographs, but becomes NP-hard for chordal graphs. On the contrary, the question whether a chordal graph can be partitioned by disjoint triangles is shown decidable in polynomial time.

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Maximum Renamable Horn Sub-CNFs

Endre Boros

A **literal** is a variable or its complement. We say, that a variable is a **positive** literal and its complement is a **negative** literal. A **clause** is a disjunction of literals with no complementary pair. A formula $\mathcal{C}$ is in **conjunctive normal form** (shortly CNF) if $\mathcal{C}$ is conjunction of clauses.

A CNF $\mathcal{C}'$ is **sub-CNF** of CNF $\mathcal{C}$ if each clause contained in $\mathcal{C}'$ is contained in $\mathcal{C}$, too. I.e. a family $\mathcal{C}'$ is subset of a family $\mathcal{C}$. A CNF $\mathcal{C}$ is called **Horn**, if each clause $C \in \mathcal{C}$ contains at most one positive literal. The **switching** of literals u and $\overline{u}$ in the CNF $\mathcal{C}$ is the operation in which every occurrence of u in $\mathcal{C}$ is replaced by $\overline{u}$ and simultaneously every occurrence of $\overline{u}$ in $\mathcal{C}$ is replaced by u. A CNF $\mathcal{C}$ is called **renamable Horn**, if there exists a subset S of variables, such that switching variables from S and their complements changes $\mathcal{C}$ to a Horn CNF.

For a CNF $\mathcal{C}$ we denote by $r(\mathcal{C})$ the size of the maximum renamable Horn sub-CNF of $\mathcal{C}$.

In this paper the NP-hard problem of finding the largest renamable Horn sub-CNF of a given CNF is considered, and a polynomial time aproximation algorithm is presented for this problem. It is shown that for cubic CNFs this algorithm has a guaranteed performance ratio of $\frac{40}{67}$. Beside this result, it is also shown that in each formula $\mathcal{C}$ you can allways find in linear time a renamable Horn sub-CNF $\mathcal{H}$ of size

$$|\mathcal{H}| = \geq \sum_{C \in \mathcal{C}} \frac{|C|+1}{2^{|C|}}.$$

(Here $|\mathcal{H}|$ is the number of clauses in $|\mathcal{H}|$ and $|C|$ is the number of literals in $|C|$.) It can be also shown, that there are formulae, for which this number is equal to $r(\mathcal{C})$.

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3-Coloring P_l free graphs in polynomial time

The coloring problem is one of the most famous and well studied NP-complete problems in graph theory. It remains hard even for various restrictions on the input graphs, e.g. planar graphs or triangle-free graphs.

In the following, if H is an induced subgraph of G, we write $H \preceq G$. A graph is called H-free if it containes no induced copy of H. A triangle or $\triangle$ is a complete graph on 3 vertices, a P_l ($l \geq 1$) is a path on l vertices. COL denotes the coloring problem, i.e. the problem of determining the chromatic number of a given graph G. The k-coloring problem, i.e. deciding whether a graph can

be colored with k colors, will be refered to as k-COL. Similarly k-SAT is the satisfyability problem for CNF-formulas containing only clauses of size at most k. The class of NP-complete problems is denoted by $\mathcal{NP}$-c.

Král', Kratochvíl, Tuza & Woeginger obtained a complete characterization of the computational complexity of COL for H-free graphs.

Theorem (Král', Kratochvíl, Tuza & Woeginger). *Let $P_3 \oplus K_1$ be the graph consisting of a P_3 and an additional isolated vertex.* COL *is polynomial time solvable on H-free graphs if $H \preceq P_4$ or $H \preceq P_3 \oplus K_1$. Otherwise* COL *remains* NP*-complete.*

Their proof of this result additionaly implies the following statement concerning the complexity of the 3-coloring problem.

Corollary (Král', Kratochvíl, Tuza & Woeginger). *If $H \npreceq P_3 \oplus K_1$ and H is no path,* 3-COL *is* NP*-complete for H-free graphs.* 3-COL $\in \mathcal{P}$ *for $H \preceq P_4$ or $H \preceq P_3 \oplus K_1$.*

So the only case remaining open is $H = P_k$ ($k \geq 4$). Since P_4-free graphs are cographs they can be colored in polynomial time. Further results for the k-coloring problem on P_l-free graphs were obtained by Sgall & Woeginger and Randerath, Schiermeyer & Tewes as displayed by the following table.

$k \backslash l$	4	5	6	7	8	9	10	11	12	...
3	$\mathcal{P}$	$\mathcal{P}$	$\mathcal{P}$	?	?	?	?	?	?	...
4	$\mathcal{P}$	?	?	?	?	?	?	?	$\mathcal{NP}$-c	...
5	$\mathcal{P}$	?	?	?	$\mathcal{NP}$-c	$\mathcal{NP}$-c	$\mathcal{NP}$-c	$\mathcal{NP}$-c	$\mathcal{NP}$-c	...

The main idea for the construction of algorithms for 3-coloring both P_5-free and P_6-free graphs dates back to a method introduced by Edwards.

The starting point is a reduction from 3-COL to 3-SAT, obtained as follows:

1. for each vertex v of the given graph $G = (V, E)$ and each color $\chi \in \{r, b, g\}$ create a variable x_v^χ
2. for $v \in V$ and $\chi, \chi' \in \{r, b, g\}$ ($\chi \neq \chi'$) add clauses assuring that not both, χ and χ', get assigned to v to the 3-SAT-formula $\mathcal{F}$ under construction
3. for $vv' \in E$ add clauses preventing v and v' from obtaining the same colors
4. for $v \in V$ add a clause stating that v actually gets some color.

Observe that all clauses of $\mathcal{F}$ but those added in step 4 are 2-clauses. In the case that the structure of the input graphs allows us to construct a formula equivalent to $\mathcal{F}$ where all the 3-clauses are replaced by 2-clauses the problem reduces to a 2-SAT instance. The observation of Edwards was now, that this idea can be applied if a constant size dominating set D is given for the graph $G = (V, E)$ under consideration. Indeed, for each fixed coloring χ of D and any vertex $v \in V \setminus D$ the list of colors that can be assigned to v in a valid extension of χ to G is of size at most two. Therefore the 3-clauses added in step 4 of the reduction can be replaced by 2-clauses. Thus 3-COL can be decided for G by solving one 2-SAT problem for every coloring of D. Since 2-SAT $\in \mathcal{P}$ this reduction yields a polynomial time algorithm for deciding 3-COL.

Nothing is known about the complexity of 3-COL for P_l-free graphs for $l \geq 7$.

Question. *Is there an l such that for all $i > k$ the* 3*-coloring problem on P_i-free graphs is* NP*-complete but for $i \leq l$ it can be solved in polynomial time.*

Two vertices are called *similar* if they are not adajacent and the neighbourhood of one of them is completely contained in the neighbourhood of the other.

Two vertices are called *similar* if they are not adajacent and the neighbourhood of one of them is completely contained in the neighbourhood of the other.

Theorem (Randerath, Schiermeyer & Tewes) Let G be a connected $\triangle, P_6$-free graph without similar vertices that is not 3-colorable. *Then G contains the Mycielski-Grötzsch graph as induced subgraph. Further on G is an induced subgraph of the* 16 *vertex Clebsch graph.*

This does not only yield a trivial algorithm for deciding 3-COL on $\triangle, P_6$-free graphs but also gives a complete structural characterization of those graphs neither containing cliques of size greater than 2 or a P_6 nor beeing 3-colorable. It would be interesting to obtain similar results for other classes of P_l-free graphs.

A bound on the chromatic number $\chi(G)$ of G with $P_l \not\preceq G$ only depending on the size $\omega(G)$ of the largest clique of G was given by Gyárfás.

Theorem (Gyárfás).

$$\chi(G) \leq (l-1)^{\omega(G)-1}.$$

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An on-line incremental approach for dynamically maintaining chordal graphs

Anne Berry, Yngve Villanger

For a chordal graph G, we study the problem of whether a new vertex u and a given set of edges between u and vertices of G can be added to G so that the resulting graph remains chordal. We show how to resolve this efficiently, and at the same time, if the answer is no, define a maximal subset of the proposed edges that can be added, or conversely a minimal set of extra edges that should be added in addition to the given set. In order to do this, we give a new characterization of chordal graphs and, for each potential new edge (u, v), a characterization of the set of edges incident to u that also must be added to G along with (u, v). We propose a data structure which can compute and add each such set in $O(n)$ time. Based on these results, we present a new algorithm which computes both a minimal triangulation and a maximal chordal subgraph of an arbitrary input graph in $O(nm)$ time. This time complexity matches the best known time bound for minimal triangulation, using a totally new vertex incremental approach. In opposition to previous algorithms, our process adds each new vertex without reconsidering any choice made at previous steps, and without requiring any knowledge of the vertices that might be added at further steps, which makes it the first fully on-line algorithm for these problems.

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All Trapezoids are Ramsey

Igor Kříž

A trapezoid is a quadrilateral which may be inscribed into a circle and such that two of its sides are parallel. We show that for an arbitrary trapezoid T and integer r there is an n such that any r-coloring of $\mathbf{R}^n$ contains a monochromatic copy of T.

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Note on Ramsey-type problem in geometry

Géza Tóth

It is proved that for any rectangle T and for any 2-coloring of the points of the 5-dimensional Euclidean space, one can always find a rectangle T' congruent to T, all of whose vertices are of the same color. We also show that for any

k-coloring of the $(k^2 + o(k^2))$-dimensional space, there is a monochromatic rectangle congruent to any given rectangle.

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Note on Ramsey-type problem in geometry

Géza Tóth

The investigation of Ramsey-type problems in the Euclidean space was initiated in a series of articles by Erdős et al. in 1973. Solving a problem of Erdős, Juhász proved that given any colouring of the plane by two colours (red and blue), and a four-point configuration K, one can find either two red points at distance 1 from each other or a congruent copy of K all of whose points are blue. However, Juhász also proved that this theorem does not remain true for all configurations K with at least 12 points. The aim of this note is to find a counterexample with only eight points.

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The Blind Bartender's Problem

The participant has not submitted any abstract.

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Stability and instability in discrete tomography

Andreas Alpers, Peter Gritzmann, Lionel Thorens

The discrete tomography deals with the retrieval of information about discrete objects from typically noisy data. The given data describe incidences of the

object with query sets, particularly lines. The talk presented strong instability results for the re construction task of discrete tomography for two-dimensional objects. In particular, it shows that even extremely small changes in the data may lead to entirely different solutions, which are moreover uniquely determined by these data.

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Trees of Bits and Bits of Trees

We present a data structure for online decremental connectivity on trees with O(1) time per operation. This is accomplished by speeding up a trivial O(log N) algorithm by one-level clusterization in Frederickson style where the logarithmically sized clusters are handled by misusing bit operations on integers for manipulating edge sets. Follows a paper by Stephen Alstrup et al.

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A result about the density of iterated line intersections in plane

Christopher J. Hillar, Darren L. Rhea

Let S be a finite set of points in the plane and let $T(S)$ be the set of intersection points between pairs of lines passing through any two points in S. We characterize all configurations of points S such that iteration of the above procedure produces a dense set. We also discuss partial results on the characterization of those finite point-sets with rational coordinates that generate all of Q^2 through iteration of $T(S)$.

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Graphs of Polytopes

In the beginning of the talk I introduced the definitions of polytopes, their faces, duality, graph of polytope. I stated the Steinitz theorem which says that the graph G is the graph of 3 dimensional polytope if and only if graph G is planar and 3-connected. I proved the forward implication and for the backward implication (the harder one) I sketched three proofs.

First is the classical proof using reductions of the graph. We reduce the graph to the K_4 and then show that the reductions can be reconstructed on the polytope.

The second proof is based on the "rubber band" drawing of the graph and than lifting it to the third dimension (define a convex piecewise linear function, linear on each face).

The third proof uses the Koebe's lemma which says that every planar graph has a kissing coins reprezentation (the vertices of the graph are the coins and two vertices are joined by edge if and only if the corresponding coins are touching each other). Then we use the stereographic projection to transform the coins from the plane to the sphere.

The talk was based on the lectures by Günter Ziegler.

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Various Approaches for The Number of Labeled Planar Graphs

In the first part of my talk I presented a counting technique for improving the lower bound for the number of labeled planar graphs to $27.2041^n n!$. This is joint work with Deryk Osthus and Anusch Taraz.

In the second one I showed results from Bodirsky, Groepl, Kang how to count, decompose and generate labeled outerplanar and cubic planar uniformly at random. Thereby they decompose a graph for counting. This allows them to make the correct probabilistic choices in a recursive generation of the graphs.

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On the fundamental group of finite posets

The complexity of the Constraint Satisfaction Problem (CSP) is one of the most examined questions in complexity theory. The known cases of NP-complete

CSP are those relational structures which does not admit any nontrivial idempotent operation. Feder and Vardi have proved that every CSP is polynomial-time equivalent to a poset retraction problem. The typical, known posets admitting no nontrivial idempotent operation are the crowns.

In our talk we give a combinatorial definition of the fundamental group of posets. We connect the algebraic and combinatorial properties of finite posets via a nice topological condition. This leads to a characterization of the posets from those a crown can be obtained using retracts and idempotent subalgebras.

Theorem. *The fundamental group of every idempotent subalgebra of a finite poset is trivial iff no crown is retract of an idempotent subalgebra.*

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The Plurality Problem with Three Colors

M. Aigner, G. De Marco, M. Montangero

The plurality problem with three colors is a game between two participants: Paul and Carol. Suppose we are given n balls colored with three colors. At any step of the game, Paul chooses two balls and asks whether they are of the same color, whereupon Carol answers yes or no. The game ends when Paul either produces a ball a of the plurality color (meaning that the number of balls colored like a exceeds those of the other colors), or when Paul states that there is no plurality. How many questions $L(n)$ does Paul have to ask in the worst case? We show that $3\lceil n/2\rceil - 2 \leq L(n) \leq \lceil 5n/3\rceil - 2$.

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A simple 3-sweep LBFS algorithm for the recognition of unit interval graphs

D. Corneil

A Simple Linear Time LexBFS Cograph Recognition Algorithm

A. Bretscher, D. Corneil, M. Habib, Ch. Paul

We present results of two articles providing simple so called Lexicographical breadth first search (LexBFS) algorithms for the recognition of particular classes of graphs. Namely 3-sweep LexBFS algorithm for unit interval graph recognition and 3-sweep Lex BFS algorithm for cograph recognition. Unit (or proper) interval graphs form a subclass of interval graphs (well known intersection class) with additional condition, that two no interval (representing particular vertex) contains any other interval. Cographs appear as the graphs without induced P_4. Both algorithms are linear in time with respect to the size of the representation of the graph. These algorithms are very important, as there are several other algorithms for the recognition of other classes of graphs based on LexBFS.

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Two-walks and spanning trees

The participant has not submitted any abstract.

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Some sufficient conditions for a graph to be in class 1

In this paper, we first give some upper bounds on the number of edges for two classes of planar graphs. Then using these upper bounds, we obtain some sufficient conditions for a planar graph to be of Class 1.

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The Erdős-Turán property for a class of Bases

J. Nešetřil and O. Serra

Erdős and Turán conjectured that the number of representations of n as the sum of two elements of a base A of order 2 is unbounded. In the talk, it was proved that it holds for bounded bases, more precisely:

Theorem (J. Nešetřil, O. Serra). *Let A be an asymptoic basis for $\mathbf{N}$ of order $h \geq 2$ and let $d \geq 2$. If A is (d,h)-bounded, then the number of representations of n as the sum of h elements is unbiounded (the h-Erdős-Turan property).*

Bounded basis is a special case with $h = d = 2$.

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Random regular graphs

N. C. Wormalds

We follow the paper Models of random regular graphs by N.C. Wormald, available online at

`http://www.math.uwaterloo.ca/~nwormald/papers/regsurvey.pdf.`

Given integers n, d we are interested in the probabilistic space $\mathcal{G}_{n,d}$ of all simple d-regular graphs on $\{1, 2, \ldots, n\}$, each of them having the same probability. (Simple means no loops or multiple edges.) The first two question awaiting us are:

- How to generate a random element of $\mathcal{G}_{n,d}$?
- How to compute probabilities in this model?

Both questions are answered by so-called "pairing model": Let V_d be a set of nd vertices, partitioned into n groups of d elements. Consider M—a random perfect matching (i.e. 1-regular graph) on V_d (this can be generated easily—exercise). Identify the d vertices in one group to one vertex, the resulting multigraph is denoted by $\tilde{M}$. It is easy to observe, that $\tilde{M}$ is a d-regular multigraph, and that all *simple* d-regular graphs have the same probability. This leads to the following algorithm.

1. Generate a random matching M
2. Check wheter $\tilde{M}$ is a simple graph.
3. If not, go to the first step.

With some effort it can be shown, that the probability of success is $e^{\frac{1-d^2}{4}}$ in each step, hence for a fixed d, the expected number of failures before success is constant.

The same approach can be used for computations with $\mathcal{G}_{n,d}$: instead computing directly with d-regular graphs, we count the number of corresponding perfect matchings. Consider the task of computing expected number of triangles in a random d-regular graph, suppose we for a while forget the restriction to graphs. Then we are interested in the number "pre-triangles" in a random perfect matching on V_d, by "pre-triangle" we mean three edges ab, cd, ef of the matching, such that b and c are in the same d-tuple of V_d, similarily d and e, f and a. If $f(t)$ denotes the number of matchings on t points (easily $f(t) = \frac{t!}{(t/2)!2^{t/2}}$), then the expected number of pre-triangles is

$$\binom{n}{3} 2 \binom{d}{2}^3 \frac{f(nd-6)}{f(nd)} \sim \frac{(d-1)^3}{6} .$$

With more effort it can be proved, that this is the anwer for the case of simple graphs: the expected number of triangles in $\mathcal{G}_{n,d}$ is asymptotically $\frac{(d-1)^3}{6}$. For details look in the original paper.

An interesting exercise is to prove that there is a constant $\beta < 1/2$ such that almost all d-regular graphs satisfy

$$\frac{\alpha(G)}{n} \leq \beta .$$

This may inequality has important consequences for existence of homomorphisms from cubic graphs.

For a readers convenience, we summarize some known results on random d-regular graphs (d is fixed). For credits and references to the original papers see Wormald's survey. Let $G \in \mathcal{G}_{n,d}$

- a.s. G is hamiltonian
- a.s. G is d-connected
- a.s. G has a perfect matching (d even), moreover
- a.s. G is edge-d-colorable (d even).

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Halin's Theorem for the Möbius Strip

Dan Archdeacon, C. Paul Bonnington, Marisa Debowsky, Michael Prestidge

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Halin's theorem characterizes those locally finite infinite graphs that embed in the plane without accumulation points by giving a set of six topologically-excluded subgraphs. This article gives similar lists of excluded subgraphs for those locally finite infinite graphs that embed in an open Mbius strip without accumulation points. Three respectively coarser partial orders are considered: the ray order (there are 153 minimal graphs excluded under this order), the minor order (350 graphs) and the topological order (1235 graphs). These results rely on the known obstruction sets for embedding finite graphs in the projective plane under the topological and the minor order. The relationship between these known lists and those constructed herein is similar to the relationship between Halin's graphs and $\{K_5, K_{3,3}\}$.

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Arboricities

The participant has not submitted any abstract.

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On the Graph of Generating Sets of a Simple Group

The refferred paper [1] of Igor Pak studies topics in computational group theory, especially methods of generating a random element of a given finite simple group. The main structure used for this task is a *product replacement graph*.

Let G be a finite group, and let k be greater of equal to the minimal number of generators of G. Denote $\Gamma(G,k) = (X,E)$ a graph with vertices to be generating k-tuples:

$$X = \{(g_1, \ldots, g_k) \in G^k, \langle g_1, \ldots, g_k \rangle = G\}$$

and edges representing multiplication of one coordinate in a k-tuple by the other:

$$E = \{((g_1, \ldots, g_i, \ldots, g_k), (g_1, \ldots, g_i \cdot g_j^{\pm 1}, \ldots, g_k)), 1 \leq i,j \leq k, i \neq j\}$$

Conjecture (Diaconis, Graham [2]) *Graph $\Gamma(S_n, k)$ is connected for all $n, k \geq 3$.*

Theorem 1. *Let G be a finite simple group, $k \geq 3$. Then $\Gamma = \Gamma(G,k)$ contains a connected component Γ' such that*

$$\frac{|\Gamma'|}{|\Gamma|} \to 1 \quad as \quad |G| \to \infty$$

Moreover, if $\langle g_1, \ldots, g_{k-2} \rangle = G$, then $(g_1, \ldots, g_{k-2}, id, id) \in \Gamma'$.

The proof of the theorem is based on the fact that two randomly chosen elements of a simple group almost surely generate whole the group (a famous result of Dixon, Kantor and Lubotzky, Liebeck and Shalev):

$$(*) \quad P(\langle g_1, g_2 \rangle = G, g_1, g_2 \in G) \to 1 \quad \text{as} \quad |G| \to \infty$$

Theorem 2. *With the conditions of Theorem 1, if $\langle g_1, \ldots, g_{k-1} \rangle = G$, then $(g_1, \ldots, g_{k-1}, id) \in \Gamma'$.*

The *product replacement algorithm* runs a nearest neighbor random walk on a graph $\Gamma(G,k)$ and outputs a random coordinate of the final generating k-tuple. Assume now that a finite group is simple. Let $g_1, \ldots, g_r$ be given generators. Let $k = r+1$ and $\Gamma = \Gamma(G,k)$. Consider a nearest neighbor random walk on Γ starting at $(g_1, \ldots, g_k, id, \ldots, id)$. Denote by Q_t^k the probability distribution of the random coordinate of the state of the walk after t steps. Let $Q^k = \lim_{t\to\infty} Q_t^k$. The *total variation distance* is defined as

$$||Q^k - U|| = \frac{1}{2} \sum_{g\in G} \left|Q^k(g) - \frac{1}{|G|}\right|$$

where U is a uniform distribution on G.

Theorem. *Let G be a finite simple group, $k \geq r+1$. Then*

$$||Q^k - U|| \to 0 \quad as \quad |G| \to \infty$$

We don't know if Γ is connected, but by Theorem 1 it containse a "large" connected component Γ'. Moreover by Theorem 2 we know that $\Gamma \setminus \Gamma'$ contains only minimal generating k-tuples. From $(*)$ we have $|\Gamma'| = |G|^k \cdot (1 - o(1))$ and therefore the projection on any coordinate of the generating k-tuple has (nearly) uniform distribution (in total variation distance). It is formally stated as the Theorem 3. Therefore we can sucesfully use the product replacemenet algorithm for generating of the random element of given simple group.

[1] I. Pak: *On the graph of generating sets of a simple group*, preprint, 1999.

[2] P. Diaconis, R. Graham: *The graph of generating sets of an abelian groups*, Colloq. Math. **80** (1999), 1–38.

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Introduction to Aperiodic Tilings

The world of aperiodic tilings is very fanciful and interesting. The talk offered the first meeting with this world. There was explained basic principles, examples, and methods. Among others there was shown a connection with the graph theory, particularly through a chromatic number.

Index of Speakers